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by S. E. Li, Y. Zheng, K, Li, L.-Y. Wang, and H. Zhang

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Michael Fang, Editor-in-Chief *IEEE Transactions on Vehicular Technology* 

# Platoon Control of Connected Vehicles from a Networked Control Perspective: Literature Review, Component Modeling, and Controller Synthesis

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Abstract — The platooning of connected and automated vehicles has the potential to significantly benefit the road traffic, including enhancing highway safety, improving traffic capacity, and reducing fuel consumption. This paper presents a four-component analysis framework for platoon systems from a networked control perspective, including a literature review by network awareness, unified models of key components, and two application cases for controller synthesis. The networked control perspective naturally decomposes a platoon into four interrelated components, namely, 1) node dynamics (ND), 2) information flow topology (IFT), 3) formation geometry (FG), and 4) distributed controller (DC). The existing literature is categorized under this framework and analyzed according to the component features. The unified mathematical models are derived for platoons with linear dynamics and distributed controllers. As a case study, a distributed controller synthesis method is introduced for homogeneous platoons, which guarantees the internal stability in the presence of a broad class of topologies with/without uniform time-delays. The effectiveness of the proposed method is demonstrated using numerical simulations.

*Index Terms*—Autonomous vehicle, platoon control, node dynamics, graph theory, distributed control.

#### I. BACKGROUND

The platooning of connected and automated vehicles (CAV) has recently attracted extensive research interests due to its potential to benefit the road traffic significantly, *e.g.*, enhancing highway safety, improving traffic capacity and smoothness, and reducing fuel consumption [1]-[4]. The platoon control aims to ensure that all the vehicles in a group move at the same speed while maintaining the desired space between adjacent vehicles [4].

To the best of our knowledge, the earliest implementation of

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platoon systems could date back to the PATH program in California in 1980's, where many fundamental topics were addressed, including the allocation of control tasks, design of control architectures, technologies for perception and actuation, and longitudinal/lateral control laws [2]-[6]. Since then, many attractive topics on platoon control have been discussed and studied, such as the selection of spacing policies [7]-[9], accommodation of powertrain dynamics [10], and the impact of homogeneity and heterogeneity [11]-[14]. Besides, advanced control methods have been introduced into platoon control to achieve more desirable performances. For example, Liang and Peng (1999) proposed an optimal control strategy for the upper-level controller to guarantee string stability [15]. Stankovic et al. (2000) employed the inclusion principle to decompose an interconnected platoon into locally decoupled subsystems, for which decentralized overlapping controllers were designed [17]. Barooah et al. (2009) introduced a mistuning-based control method to improve the stability margin of platoons [16]. Dunbar and Derek (2012) proposed a distributed receding horizon controller and derived the sufficient condition to guarantee string stability [18]. This method was recently extended to cover unidirectional topologies in [19]. Ploeg et al. (2014) developed an  $\mathcal{H}_{\infty}$  controller synthesis approach for linear platoons, in which string stability was explicitly satisfied by using linear matrix inequalities (LMIs) [21]. More recently, some vehicular platoons have been demonstrated under real-world settings in projects such as the GCDC in Netherlands [20], SARTRE in Europe [22], and Energy-ITS in Japan [23].

The earlier practices on platoon control often relied only on the radar-based sensing systems, which limited the types of information exchange topologies [24]. However, the rapid development of vehicle-to-vehicle (V2V) communications, such as DSRC [25], can generate various types of topologies in platoons, e.g., the two-predecessor following type and the multiple-predecessor following type [24][26][27]. New challenges arise naturally considering the variety of topologies, especially when taking into account the time delay, packet loss, and quantization error in the communications. Under such circumstances, it is preferable to view a platoon as a network of dynamical systems and to employ networked control techniques to design distributed controllers. For instance, Oncu et al. (2014) investigated the influence of network-induced constant delays and sampling-hold processes on string stability from the perspective of networked control [29]. Bernardo et al. (2014) introduced a method to analyze the problem of platoon

control from the viewpoint of consensus in a dynamic network, and proved the closed-loop stability of platoons with time-delays in the communications [31]. Wang *et al.* (2014) proposed a weighted and constrained consensus seeking framework to analyze the influence of time-varying network on platoon dynamics via a discrete-time Markov chain model [30]. Zheng *et al.* (2016) explicitly derived the stable region of linear feedback gains for platoons with a broad class of topologies by using the Routh-Hurwitz stability criterion [32]. Zhang and Orosz (2016) investigated the effects of heterogeneous connectivity structures and information delays on platoon systems [33]. The stability margin of homogeneous platoons with typical topologies was studied using matrix factorization and eigenvalue analysis in [34]-[39].

From the perspective of networked control, a platoon with multiple connected and automated vehicles is actually a one-dimensional network of dynamical systems, in which each node only uses its neighboring information for control and aims to achieve certain global coordination with all other nodes. Such perspective naturally decomposes a platoon into four interrelated sub-components, namely [32][37],

- 1) Node dynamics (ND),
- 2) Information flow topology (IFT),
- 3) Formation geometry (FG),
- 4) Distributed controller (DC).

This decomposition provides a unified four-component framework to analyze, design, and synthesize a vehicular platoon, as well as the on-road implementation of control strategies [32][37]-[39]. The main purpose of this paper is to present a literature review of existing platoon techniques, and to introduce a generic modeling framework for generic platoons. Moreover, as a special case, a distributed controller synthesis method is proposed to address the diversity of topologies and the existence of time-delays on homogeneous platoons. The major contributions are:

(1) This paper summarizes the existing outcomes on platoon control by literature categorization and technical analysis. Also, we summarize the major techniques on how to deal with three performance metrics, *i.e.*, string stability, stability margin, and coherence behavior.

(2) Under the proposed four-component framework, this paper provides unified models for key components by employing the feedback linearization technique and algebraic graph theory.

(3) A distributed controller synthesis approach is proposed for homogeneous platoons with/without uniform time-delays, which converts the controller design into the solutions to a parametric algebraic Riccati equation. This technique is able to guarantee the internal stability of platoons in the presence of a broad class of topologies.

The remainder of this paper is organized as follows. Section II presents a brief introduction of the four-component framework. Section III reviews platoon control methodologies and the techniques on addressing typical performance metrics. The modeling process for the four main components is described in Section IV. Section V derives the closed-loop dynamics for typical homogeneous platoons, and proves two theorems on the stabilization of distributed controllers. This

section also includes simulation results to demonstrate the effectiveness of our design method. Section VI concludes the paper with some remarks. A preliminary version of this paper was presented in [38], and the material of this paper was partially summarized in [37].

#### II. INTRODUCTION TO FOUR-COMPONENT FRAMEWORK

As shown in Fig. 1, we consider a platoon running on a flat road with N + 1 vehicles, including a leading vehicle (LV, indexed by 0) and N following vehicles (FVs, indexed from 1 to N). A platoon system can be viewed as a composition of four main sub-components, *i.e.*, 1) node dynamics; 2) information flow topology; 3) formation geometry, and 4) distributed controller, which is referred to as the four-component framework. This framework is demonstrated in Fig. 1, where each component is defined as follows.

- 1) Node dynamics (ND), which models the longitudinal response of each vehicle in the platoon;
- 2) Information flow topology (IFT), which specifies how a node obtain information about other nodes;
- 3) Formation geometry (FG), which defines the desired spacing between adjacent vehicles in a platoon.
- 4) Distributed controller (DC), which implements feedback control using the information specified by IFT;



Fig. 1 Four major components of a platoon [32][37]: 1) node dynamics, 2) information flow topology, 3) distributed controller, 4) formation geometry; where  $d_r$  is the actual relative distance,  $d_{des}$  is the desired distance,  $u_i$  is the the control signal for *i*-th vehicle, and  $C_i$  denotes the controller installed in *i*-th vehicle.

The internal stability is the first priority of all platoons. A platoon with linear time-invariant dynamics is internally stable if and only if real parts of the eigenvalues of the closed-loop system are all strictly negative [35]. In addition to internal stability, other performance metrics for a platoon include 1) string stability; 2) stability margin, and 3) coherence behavior:

*Definition* **1** (*String Stability*). A platoon is string stable if the disturbances are attenuated when propagating downstream along the string [11][40][46];

**Definition 2** (Stability Margin). The stability margin is defined as the absolute value of the real part of the least stable eigenvalue, characterizing the speed of convergence for the initial errors in a platoon [35][36];

**Definition 3 (Coherence Behavior).** The coherence behavior is quantified as the  $\mathcal{H}_2$  norm of the closed-loop system [59][75], capturing the robustness of a platoon with respect to exogenous white noises.

Each sub-component in Fig. 1 can exert significant influence on the performance metrics of a platoon, which will be discussed in Section III. The details of modeling processes for these four components will be described in Section IV. According to the features in each component, the existing literature on platoon control is categorized into several groups, as listed in TABLE I.

TABLE I.	CATEGORIZATION OF PLATOON CONTROL
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		Noc	le Dynamics (ND	)						
Single-integrator	Second-	-order model	Third-ord	SISO model	l Nonline	ear model				
[57][58][59][60]	[13][15][16][30][31][35][36][47]		[3][5][8][9][10][	11][14][15][17]	[6][12][46]	[18][19	][40][48]			
[61]	[49][54][55][54	6][57][67][70]-[72]	[20][21][24][28]	[29][32][39][41]	[63][68][73]	[62][64	][69][88]			
	[75][78][81][82][87]			[50][51][76][77][79][80][83][85][86]						
	Homo	geneous			Heterogeneo	us				
[3][5][6][8][10][15][16][21][24][30][32][35][36][39][46][47			][51][54][56][57][ [9][10][11][12][14][17][18			)[28][29][31	][40][41][			
58][59][60][61][63][	67][68][70][71][7	72][74][75][78][79][82]	[83][84][87][88]	48][49][50][55][62	2][64][69][73][7	6][77][80][8	1][85][86]			
Information Flow Topology (IFT)										
PF		PLF	BI	)	BDL	,	TPF			
[3][8][9][10][11][15]	[20][21][24] [	5][6][11][14][17][18][	[12][13][16][24	][35][36][39]	[12][24][32][39	[21]	[24][32]			
[29][41][46][47][50][	54][56][62] 24	4][28][40][46][64][67]	[46][48][51][55	5][56][57][68]		[6	7][73]			
[67][70][74][79][80][82][85][86] [76][78][83] [69][73][77][81][82][84][87]										
TPLF		Undirected	Limited Rang	ge	General Top	ologies				
[24][32] [24][32][39][49][51][71][88]			[73][75] [19][24][30][31][32][35][58][59][60][61][63][72				1][63][72]			
Formation Geometry (FG)										
CD CTH NLD						NLD				
[3][5][6][9][11][12][13][14][16][17][18][19][30][24][28][32][35][36][39][40][46][48][49] [9][10][15][20][21][29][31][41][4 [8][79]					[8][79]					
	13][14][10][17][.	18][19][30][24][28][32]	[33][30][39][40][40	1[][]	[51][54][55][56][57][58][59][60][61][63][64][67][68][70][71][72][73][75][76][77][78] 7][50][62][69][73][74][80][82]					
[51][54][55][56][5	7][58][59][60][61	l][63][64][67][68][70][7	71][72][73][75][76][	77][78] 7][50]	][62][69][73][74	][80][82]				
[51][54][55][56][5	7][58][59][60][61 [81][8	18][19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88]	71][72][73][75][76][	77][78] 7][50]	][62][69][73][74 [85][86]	][80][82]				
[5][5][54][55][56][5	7][58][59][60][61 [81][8	18][19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib	uted Controller (	<b>DC</b> )	][62][69][73][74 [85][86]	.][80][82]				
[51][54][55][56][5	7][58][59][60][61 [81][8 Linear Contro	18][19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib ller	uted Controller (	$\frac{\mathbf{DC}}{\mathbf{DC}}$	][62][69][73][74 [85][86] controller	.][80][82] SMC	MPC			
[3][6][9][11][12][13]	[15][14][10][17][ 7][58][59][60][61 [81][t [81][t Linear Contro [16][20][24][29][	18][19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib 1ler [30][31][32][35][36][39]	uted Controller ( Optimal Co [4][40][40][40] [7][72][73][75][76][ Optimal Co	<b>DC</b> ) $\mathcal{H}_{01}(3) = [2](3)$ $\mathcal{D}_{01}(3) = [2](3)$ $\mathcal{D}_{01}(3)$	][62][69][73][74 [85][86] controller [[21][28]	[80][82] SMC [5][8][10]	MPC [18][19]			
[3][6][9][11][12][13] [3][6][9][11][12][13] 1][46][47][49][54][55]	[15][14][10][17][ 7][58][59][60][61] [81][t Linear Contro [16][20][24][29][ ][56][58][60][61]	18][19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib ller [30][31][32][35][36][39] ][63][67][68][69][70][7	uted Controller ( Optimal Cc [4][40][40][40][40] [7][72][73][75][76][ Optimal Cc [4 [15][17][57] [1][ [72]	$\frac{DC}{[59][62]} = \frac{\mathcal{H}_{\infty}}{[59][62]} = \frac{\mathcal{H}_{\infty}$	][62][69][73][74 [85][86] controller [[21][28] [1][76] [4	][80][82] SMC [5][8][10] 40][48][64]	MPC [18][19] [20][50]			
[3][6][9][11][12][ [51][54][55][56][57 [3][6][9][11][12][13] 1][46][47][49][54][55 73][74][75][77][7	Linear Contro [16][20][24][29][61] [16][20][24][29][ [16][56][58][60][61] 8][79][80][81][83]	18[[19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib 1ler [30][31][32][35][36][39] ][63][67][68][69][70][7 2][83][84][85][86][87]	uted Controller ( Optimal Cc [4][40][40][40] uted Controller ( Optimal Cc [4 [15][17][57] [1][ [72]	$\frac{DC}{59][62]} = \frac{\mathcal{H}_{\infty}}{59}$	][62][69][73][74 [85][86] controller [[21][28] [1][76] [4	][80][82] SMC [5][8][10] 40][48][64] [88]	MPC [18][19] [20][50]			
[3][6][9][11][12][ [51][54][55][56][57 [3][6][9][11][12][13] 1][46][47][49][54][55 73][74][75][77][7	Linear Contro [16][20][24][29][ [16][20][24][29][ [56][58][60][61 8][79][80][81][8:	18[[19][30][24][28][32] 1][63][64][67][68][70][7 82][83][84][87][88] Distrib iler [30][31][32][35][36][39] 1[63][67][68][69][70][7 2][83][84][85][86][87] Pla	uted Controller (         Optimal Co         ][4         [15][17][17][57]         [16][17][17][17]         [17][17][17][17]         [17][17][17][17]         [17][17][17][17]         [17][17][17][17]         [17][17][17][17][17]         [17][17][17][17][17][17][17]         [17][17][17][17][17][17][17][17][17][17]	$\begin{array}{c} \mathbf{DC} \\ \mathbf{DC} \\ \text{ontroller} & \mathcal{H}_{\infty} \text{ or} \\ [59][62] & [14] \\ [59] \end{array}$	][62][69][73][74 [85][86] controller [[21][28] [1][76] [4	][80][82] SMC [5][8][10] [48][64] [88]	MPC [18][19] [20][50]			
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# III. REVIEW OF PLATOON CONTROL FROM THE FOUR-COMPONENT FRAMEWORK

In this section, we categorize the existing literature into different groups according to the features of their components. Three performance metrics are discussed, namely, 1) string stability, 2) stability margin, and 3) coherence behavior. Also, we give a brief discussion on the communication issues of platoon control.

#### A. Summary of Each Component

# 1) Node Dynamics (ND)

Most of previous studies on platoon control only emphasizes on the longitudinal dynamical behaviors of ND. Only a few studies discussed the integrated longitudinal and lateral control (see, *e.g.*, [64][65]). Bicycle model is usually used to describe the lateral dynamics for control design (see [64][65] for detailed descriptions). Here, we mainly review the modeling of ND in longitudinal direction.

The vehicle longitudinal dynamics are inherently nonlinear

due to some salient nonlinearities involved in the powertrain system, *e.g.*, engine, driveline, brake system, aerodynamics drag [18][66][94]. Some studies directly employed nonlinear models in the analysis and design of platoons (see [18][40][48] [64] for example). The asymptotic stability and string stability can be guaranteed by carefully adjusting the control parameters. Explicit performance limit, however, is rather difficult to analyze given spacing policies and communication topologies for nonlinear models. In fact, linear models are more frequently used to formulate tractable problems. The most commonly used models are 1) single integrator model, 2) second-order model (including double-integrator model), 3) third-order model, and 4) single-input-single-out (SISO) model (see TABLE I. for the categorizations).

The single integrator model is the simplest case, where the vehicle speed is taken as the control input and the position is the exclusive state. This model can significantly simplify the theoretical analysis on certain controller synthesis. For instance, the design of optimal controller for platoons can be equivalently reduced into a structured convex optimization problem under the assumption of single integrator, while this problem is quite challenging for other types of models [57][59]. However, because of departing substantially from actual vehicle dynamics, the single integrator model fails to reproduce the string instability phenomena [49]. One improvement on modeling is to consider ND as a point mass, leading to the double-integrator model [15][16][49], or to view a platoon as a mass-spring-damper system, resulting in the linear secondorder model [78][81][82]. Both of these two models use acceleration as the control input, which are the basis of many important theoretical results, such as decentralized optimal control [57], scaling trend of stability margin [16][35][36][55], and coherence behavior [75]. Note that the assumption of directly controlling the acceleration still does not capture some features of vehicle internal dynamics, e.g., inertial delay in powertrain dynamics, and might lead to instability in real world driving conditions [10][17][20][21]. One solution is to further increase one state to yield the so-called third-order model. The increased state approximates the powertrain's input/output behaviors, which is equivalent to degrade the control input to engine torque and braking torque [8][10][14][17][21]. Now, most approximation methods employ either the feedback linearization technique [3][10][17][24] or the lower-layer control technique [2][4][64][94]. The last, but not the least, class of models is the so-called SISO model, which is often adopted to analyze string stability in the frequency domain. The pioneering work on this model started from Seiler, Pant and Hedrick [46], which was widely used in many other studies; see e.g., [12][63][68][73] and [74].

Multiple vehicles are involved in a platoon, and thus an important feature for ND is homogeneity. A platoon is said to be "homogeneous" if all vehicles share identical dynamics; otherwise, it is called "heterogeneous". The assumption of homogeneous platoons can greatly simplify the theoretical analysis of platoon control (see [16][21][24][35]), while the heterogeneous assumption is more aligned with the realistic conditions (see [9]-[12][17]-[20][73]).



Fig. 2 Typical IFTs for Platoons. (a) PF; (b) PLF; (c) BD; (d) BDL; (e) TPF; (f) TPLF

# 2) Information Flow Topology (IFT)

The IFT in a platoon has close relationship with the way a vehicle obtains the information of its neighboring vehicles. The IFT describes the information used by the local controller in each node, and thus has significant influence on the collective behaviors of a platoon, *i.e.*, string stability [46], stability margin [32][35] and coherence behavior [57][58].

Early-stage platoons are mainly based on radars to sensor the

surrounding conditions, which means that a vehicle can only acquire the information of its nearest neighbors, *i.e.*, front and back ones. Using this kind of sensing system, the typical IFTs are the predecessor-following (PF) and the bidirectional (BD) types. With the rapid development of V2V communications, many other types of IFTs are emerging, such as the two predecessor-following (TPF) type, two predecessor-leader following (TPLF) type, undirected type, and limited communication range topologies (see TABLE I. for a categorization). Some typical IFTs for platoons are shown in Fig. 2. Note that the results of TABLE I. are only based on the connection characteristics among communications, and no communication characteristics such as quantization errors, data dropouts and time-delays are considered here. These issues have been recently discussed in a broader sense of networked control systems. For example, Zhang and Wang (2014) proposed a stabilizing methodology for networked control systems subject to both feedback and forward communication delays [91]. The filtering problem of discrete-time Takagi-Sugeno (T-S) fuzzy systems was investigated in a network environment, where multiple packet dropouts were considered by using Markov chains [92][93]. Please refer to Section III.C for further discussions.

No matter what kind of topologies is adopted in a platoon, internal stability must be guaranteed. Typically, there exist two main approaches to ensure the internal stability: 1) global approach [14][76][111], and 2) local approach [8][10][21][29]. The first approach straightforwardly takes the whole platoon as a structured system, and then designs the controller in a centralized way, where the IFT becomes less important in design process. For instance, several linear matrix inequalities (LMIs) were derived to guarantee internal stability based on the global platoon dynamics in [14][76][111]. One major drawback for this approach is on the computation efficiency, which would quickly worsen as the increase of the platoon size. Thus, most studies tried to decompose a platoon into multiple sub-systems, and design the controller in a decentralized way, resulting in the second approach. For example, under PF topology type, a platoon can be naturally considered as a unidirectional cascade system, which only needs to focus on any two successive vehicles to guarantee stability of a platoon [8][10][21][29][41]. Additionally, the inclusion principle was employed to decompose this kind of platoon into multiple locally decoupled subsystems, where overlapping controllers could be designed [17]. However, this decomposing technique is not universal. For example, it does not suit for the BD topology because its spacing errors propagate from both forward and backward directions. Partial differential equation (PDE) technique was used to approximate the dynamics of platoons under BD topology in [13][16][35][36][77], which could avoid the difficulties of analyzing high dimensional dynamics. Besides, for homogeneous platoons under more general topologies, the similarity transformation and matrix factorization is an important approach, which decomposes the internal stability into two parts: 1) stability of information flow for the given IFT and 2) stability of individual vehicles for the given DC [24][32] [63]. Note that this approach does not work for heterogeneous

platoons in general. More recently, chordal decomposition techniques have been applied to the design of structured controllers, which are promising to deal with heterogeneous vehicle platoons [105][106].

### 3) Formation Geometry (FG)

The objective of platoon control is to track the speed of the leading vehicle while maintaining a formation governed by the desired spacing policy between any two consecutive vehicles. In general, there are three main policies of FG employed in platoons, namely 1) constant distance (CD) policy, 2) constant time headway (CTH) policy, and 3) nonlinear distance (NLD) policy [7][67]. For the CD policy, the desired distance between two adjacent vehicles is constant and independent of vehicle's velocity, which can achieve a very high traffic capacity. However, more attentions are needed on the controller parameters and communication connections to guarantee string stability or certain stability margin when using CD policy [32] [39]. For the CTH policy, the desired inter-vehicle spacing varies with vehicle's velocity, which is more likely in accordance with driver behaviors, but has limit on achievable traffic capacity. It is known that there exist minimum time headway requirements such that certain platoons is string stable [74]. For the NLD policy, the desired inter-vehicle distance is a nonlinear function of vehicle's velocity, which has the potential to balance the traffic flow stability and traffic capacity compared with CD and CTH policies (see [8][79] for details).

# 4) Distributed Controller (DC)

The DC implements the feedback control using neighboring information to achieve certain global coordination, which can be either structured or unstructured. An unstructured DC is equivalent to a complete graph which allows communications between all pairs of vehicles. Many existing studies belongs to structured control in an either explicit or implicit way (see [4][9] [10][17][21][35]). The structural property is determined by IFTs, which brings both the difficulties in controller design [57] and fundamental limitations in platoon performances [32]-[35].

The majority of DCs are linear for the easiness of comprehensive theoretical analysis, and the convenience in hardware implementation [7][11][16][24]. The internal stability of platoons with linear controllers largely depends on the structure of IFTs, which means the design of linear DC is often case-by-case based. For instance, the stabilizing region of linear control gains was explicitly derived for a large class of topologies in [24], and the string stability requirements for platoons under the PF topology were established in [10]. Some optimization methods, either numerical or analytical ones, were proposed to optimize the localized gains in [15][17][57].

There are two major drawbacks in linear design methods, namely 1) it is not easy to explicitly handle string stability, and 2) it is unable to deal with the nonlinearity and constraints. Recently, advanced control methods have been introduced into platoon control for achieving better performances. For example, sliding mode control (SMC) was used to design string-stable platoons [5][8][10], where internal stability and string stability were realized by a posterior controller tuning. The  $\mathcal{H}_{\infty}$ controller synthesis was proposed to include the string stability requirement as *a priori* design specification in [21]. Model predictive control (MPC) technique was also introduced into platoon control community, which could explicitly handle the node nonlinearities and actuator constraints by formulating multiple local convex problems defined in the predictive horizon [18]-[20][50]. Note that there also exist advanced observer design methods for vehicle systems, such as  $\mathcal{H}_{\infty}$ analysis [51][52] and adaptive sliding-mode method [53][88].

### B. Summary of Platoon Performance

Some practical benefits by platooning, *e.g.*, reducing fuel consumption and improving traffic efficiency, are out of the scope of this paper. We refer the interested reader to [42]-[45] for discussions on these topics. The major techniques for internal stability are summarized in Section III.A. Hence, here we focus on the discussions on string stability, stability margin and coherence behavior.

# 1) String Stability

Internal stability of a platoon in the Lyapunov sense does not naturally lead to string stability. If it is not well designed, error signals can be amplified when propagating downstream the string even the closed-loop system is internal stable, which may eventually result in rear-end collision [10][46]. This phenomenon is called string instability, *e.g.*, in [46][73], or slinky effect, *e.g.*, in [17].

The achievability of string stability is closely related to choices of FGs and IFTs in a platoon. For instance, Seiler *et al.* (2004) proved that due to the complementary sensitivity integral constraint, string stability cannot be guaranteed for any linear identical controllers under the PF topology and CD policy [46]. This is a fundamental phenomenon and independent with the choices of controller gains for such kind of platoons. Barooah *et al.* (2005) further pointed out that linear identical controllers also suffered fundamental limit on the string stability for a homogeneous platoon under the BD topology [68]. Recently, Middleton *et al.* (2010) extended the work in [46] to cover limited communication range and non-zero time headway policy [73]. It was shown that both forward communication range and small time headway could not alter the string instability.

Some methods have been proposed to improve string stability, which can be categorized into three groups.

- a) Relax formation rigidity, *i.e.*, introducing enough time headway in the spacing policy (*e.g.*, [9][10][15]), or adopting NLD policy (*e.g.*, [8][79]);
- b) Use non-identical controllers, *i.e.*, choosing difference controller parameters for different nodes (*e.g.*, [16][54]);
- c) Extend the information flow, *e.g.*, broadcasting the leader's information [11][17]. It is pointed out that it is necessary to have some global information (*e.g.*, leader's velocity) to ensure string stability when using CD policy [72].

Recently, various types of controllers have been proposed to ensure string stability in the literature, including SMC [8][10], MPC [18][20] and  $\mathcal{H}_{\infty}$  controller [21][76]. Note that these controllers either employed the CTH policy or used certain global information.

#### 2) Stability Margin

Stability margin is a performance index to characterize the convergence speed of initial errors in a platoon [35][36]. Most of existing research on stability margin focuses on the CD policy, which has revealed that stability margin is a function of 1) platoon size (N), 2) ND, 3) IFT, and 4) the DC structure [16][32]-[39].

By considering ND as a point mass, Barooah et.al (2009) demonstrated that the stability margin would approach zero as  $O(1/N^2)$  with the increase of platoon size for symmetric bidirectional control, and proved that asymptotic behavior of stability margin could be improved to O(1/N) by introducing certain "mistuning" [16]. This result was extended to linear third-order dynamics that could cover the inertial delay of powertrain dynamics in [32]. In addition, using partial differential equation (PDE) approximation, Hao *et al.* (2011) showed that the scaling law of stability margin could be improved to  $O(1/N^{2/D})$  under D-dimensional IFTs [35]. In [36], it was proved that employing asymmetric control, the stability margin could be bounded away from zero, which was independent of the platoon size N. Zheng et al. (2016) further introduced two basic methods to improve the stability margin from the perspective of topology selection and control adjustment [39].

### 3) Coherence Behavior

The coherence behavior is the  $\mathcal{H}_2$ -norm of the closed-loop system. This index is to character the robustness of platoons driven by exogenous white noises, which captures the notion of coherence [61][75]. Bamieh *et al.* (2012) investigated the asymptotic scaling trend of the upper bounds on coherence behavior with respect to platoon size, and indicated the structure of IFTs may play a more important role than the parameters of DC [75]. Besides, several recent articles adopted coherence behavior as the cost function to optimize the local control gains [57], and the communication structure of IFTs [58][59][61], via the augmented Lagrangian approach and alternative direction method of multipliers (ADMM). We note that chordal decomposition techniques in [105][106] might be promising to deal with the coherence behavior as well.

# C. Communication Issues of Platoon control

Wireless communication enables a platoon to exchange information between vehicles in addition to local sensor measurements, which is highly potential to improve platoon performances [25][29][39]. However, inter-vehicle wireless communication usually introduces network imperfections, such as transmission time-delays, sampling intervals, packet loss [108]. It is critical to take communication time-delays and other network-induced effects into account when analyzing and designing a platoon system in practice. In this section, we give a discussion on the strategies that deal with time-delays and other network-induced effects.

# 1) Time-delays

The effect of time-delays in wireless link on vehicle platoons already attracted considerable attention in the past. For example, Liu *et al.* pointed out that string stability could be compromised

by communication delays, where an upper bound on the delay of preceding vehicle information was given to maintain string stability [5]. Moreover, a field test consisting of six passenger cars was carried out in [89], which clearly demonstrated that the existence of communication delays would compromise string stability. In [9], Naus et al. derived a necessary and sufficient condition in the frequency domain to guarantee string stability, considering both communication delays and heterogeneous dynamics. More recently, Bernardo et al. introduced a method to convert a platooning problem into a problem of achieving consensus in a network of dynamic systems, which could handle time-varying heterogeneous delays [31]. Gao et al. presented an H-infinity control method for a platoon of heterogeneous vehicles, where both uncertain vehicle dynamics and uniform communication delay were considered [111]. In the case where communication link has increased time-delay or even suffers persistent link failure, Ploeg et al. proposed a control strategy for graceful degradation of one-vehicle look-ahead platoons, using onboard sensors to estimate the preceding vehicle's acceleration [90]. In the Grand Cooperative Driving Challenge (GCDC), the information of vehicles was sent at an update rate of 10 Hz, which means time-delay was an inevitable factor to be addressed in the design and experimental validations [20][80].

In a practical platoon, there also exists certain vehicle actuator delays in addition to communication delays. Some experimental results have shown that the actuator delay cannot be ignored [47][89]. We note that some recent work has considered actuator delays when designing a practical platoon system; see, *e.g.*, [20][47][89][90]. In [10], Xiao and Gao derived a minimum time-headway gaps to guarantee string stability for both homogenous and heterogenous platoons, where the time delays of actuators and sensors were considered.

In theory, it in general leads to a non-rational transfer function when considering time-delays in communications or actuators, which poses a challenge for the controller design. One useful method is to approximate the time-delays using Padé approximation [112]. For example, the actuator delay was approximated by a second-order Padé approximation in [20], and a third-order Padé approximation was used to approximate the communication delay in [21]. More recently, Xing *et al.* proposed a procedure to find the lowest order of Padé approximation for the delays in vehicle platoons [112].

# 2) Other Network-induced Effects

Wireless communication is essentially probabilistic. In addition to time delays, wireless link also introduces other network imperfections, such as uncertain sampling intervals and packet loss. Some recent efforts have been made to investigate the impact of communications on the general networked control systems, which can be categorized into several main issues [108][109]: (1) additive noises, which follows traditional physical layer models; (2) communication latency, which is modeled as a maximum delay or random delays with certain distributions; (3) packet loss, which is characterized by packet delivery ratio; (4) asynchronous data flow, which describes asynchronous patterns of transmitting or receiving signals among different channels.

These issues imply that communication systems for platoons may introduce irregular and asynchronous information flow patterns, which have significant impact on platoon performance [109][113]. For example, Hafeez et al. proposed an analytical model to address the reliability of dedicated short-range communication (DSRC) systems in vehicular ad hoc networks (VANETs) [110]. Xu et al. investigated the coordinated control and communication design of a platoon by employing TCP-based communication protocols, in which the detrimental effects of random delays on vehicle safety were considered [117]. In a generic framework, the impact of communications can be treated by viewing communication systems as added uncertainties and constraints [116][119]. The results of this direction could be applied to platooning problems with the potential to further improve the performance of a practical platoon [30]. From a networked control system perspective, Oncu et al. addressed the influence of network-induced constant delays and sampling-hold processes on string stability, where the model-based analysis results were validated by real experiments [29]. Recently, Guo and Wen introduced a framework for network access scheduling and platoon control co-design, which could deal with capacity limitation and random packet dropouts [114].

In addition, considering the sharing of communication resources between interfering vehicles, it is important to enable predictable allocation of real-time communication capacity among interfering vehicles. Existing IEEE 802.11p-based inter-vehicle wireless communication does not enable predictable control of co-channel wireless interference [118]. The current practice adopted by the automotive industry reduces information exchange rate to avoid severe interference. Without addressing the fundamental problem of interference control, however, this approach increases communication delay and reduces communication throughput. Thus, it is important to investigate mechanisms for ensuring predictable interference control and wireless networking in general, which is essential to the co-design of control polices and communication systems for practical platoon systems [120].

#### IV. UNIFIED MODELING FOR THE FOUR COMPONENTS

This section presents the modeling process for the four main components, *i.e.*, 1) model of node dynamics, 2) directed graph for information flow topology, 3) desired spacing policy, and 4) design of distributed controller.

### A. Model for Node Dynamics

The longitudinal dynamics are inherently nonlinear, which include engine, drive line, brake system, aerodynamics drag, tire friction, gravitational force. For the sake of simplicity, some reasonable assumptions were used to obtain a concise model for platoon control [17][24][66]: 1) The longitudinal tire slip is negligible, and the powertrain dynamics are lumped into a first-order inertial transfer function; 2) The vehicle body is rigid and symmetric; 3) The influence of pitch and yaw motions is neglected; 4) The driving and braking torques are controllable inputs.

The longitudinal vehicle dynamics are simplified, but still nonlinear, as follows:

$$\begin{cases} \dot{p}_{i}(t) = v_{i}(t) \\ \frac{\eta_{T,i}}{r_{w,i}} T_{i}(t) = m_{i} \dot{v}_{i}(t) + C_{A,i} v_{i}^{2}(t) + m_{i} g f , & i \in \mathcal{N}, \\ \tau_{w,i} \dot{T}_{i}(t) + T_{i}(t) = T_{des,i}(t) \end{cases}$$
(1)

where,  $\mathcal{N} = \{1, 2, \dots, N\}$ ;  $p_i(t)$  and  $v_i(t)$  denote the position and velocity of vehicle i;  $m_i$  is the vehicle mass;  $C_{A,i}$  is the lumped aerodynamic drag coefficient; g is the acceleration of gravity; f is the coefficient of rolling resistance;  $T_i(t)$  denotes the actual driving/braking torque;  $T_{des,i}(t)$  is the desired driving/braking torque;  $\tau_i$  is the inertial delay of vehicle longitudinal dynamics;  $r_{w,i}$  denotes the wheel radius and  $\eta_{T,i}$  is the mechanical efficiency of driveline. The position and velocity of the leading vehicle are denoted by  $p_0(t)$  and  $v_0(t)$ , respectively.

The exact feedback linearization technique is often used to convert the nonlinear model into a linear one for controller design. The same technique has been widely used before [10] [14][17][77]. The exact feedback linearization law is as follows:

$$T_{des,i}(t) = \frac{1}{\eta_{T,i}} \Big( C_{A,i} v_i (2\tau_i \dot{v}_i + v_i) + m_i g f + m_i u_i \Big) r_{w,i},$$
(2)

where  $u_i$  is the new input signal after linearization. Then, a linear model is obtained for vehicle longitudinal dynamics:

$$\tau_i \dot{a}_i(t) + a_i(t) = u_i(t), \tag{3}$$

where  $a_i(t) = \dot{v}_i(t)$  denotes the acceleration of vehicle *i*.

Then, a 3<sup>rd</sup>-order state space model is derived for each vehicle:

$$\dot{x}_{i}(t) = A_{i}x_{i}(t) + B_{i}u_{i}(t)$$

$$x_{i}(t) = \begin{bmatrix} p_{i} \\ v_{i} \\ a_{i} \end{bmatrix}, A_{i} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_{i}} \end{bmatrix}, B_{i} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_{i}} \end{bmatrix},$$
(4)

where  $x_i(t) = [p_i, v_i, a_i]^T$  is the state of node *i*.

This linearized  $3^{rd}$ -order model has been widely used in the literature as a basis for theoretical analysis, *e.g.*, [9]-[11][15] [17][20][21][24]. In addition, there are some studies assuming the vehicle dynamics as ideal double integrators [13][16][35] [36], which neglects the inertial delay in powertrain dynamics:

where the control input  $u_i(t)$  for this model is the acceleration of each vehicle.

The simplest model for node dynamics is the single integrator, *e.g.*, [57]-[61],

$$\dot{p}_i(t) = u_i(t). \tag{6}$$

where the control input  $u_i(t)$  is the velocity of each vehicle.

# B. Model for Information Flow Topology

We use directed graphs to develop a unified model for allowable information flow interconnecting the vehicles in a platoon. More descriptions on graph theory can be found in [95] This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2017.2723881, IEEE Transactions on Vehicular Technology

and the references therein.

A platoon includes *N* followers and one leader. The information flow among followers is described by a directed graph  $\mathcal{G}_N = \{\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N\}$  with a set of *N* nodes  $\mathcal{V}_N = \{1, 2, \dots, N\}$ , a set of edges  $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$  and the adjacency matrix  $\mathcal{A}_N = [a_{ij}] \in \mathbb{R}^{N \times N}$ . Node *i* represents the *i*-th follower in a platoon, and each edge (j, i) in  $\mathcal{E}_N$  represents a directional information flow from vehicle *j* to vehicle *i*. An edge (j, i) belongs to set  $\mathcal{E}_N$ , *i.e.*,  $(j, i) \in \mathcal{E}_N$  if and only if  $a_{ij} = 1$ , which means vehicle *i* can receive the information on vehicle *j*; otherwise  $a_{ij} = 0$ . It is assumed that self-edges (i, i) are not allowed, *i.e.*,  $(i, i) \notin \mathcal{E}_N$  for all  $i \in \mathcal{V}_N$ . The neighbor set of node *i* is denoted by  $\mathbb{N}_i = \{j | a_{ij} = 1\}$ . The in-degree of *i*-th vehicle is represented by  $\deg_i = \sum_{j=1}^N a_{ij}$ . Denote  $\mathcal{D}_N = diag\{\deg_1, \deg_2, \cdots, \deg_n\}$ , and the Laplacian matrix  $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$  of  $\mathcal{G}_N$  is defined as  $\mathcal{L} = \mathcal{D}_N - \mathcal{A}_N$ .

To model the information flow from the leader to the followers, we define an augmented graph as  $\mathcal{G}_{N+1}$  with a set of N + 1 nodes  $\mathcal{V}_{N+1} = \{0, 1, 2, ..., N\}$  and a set of edges  $\mathcal{E}_{N+1} \subseteq \mathcal{V}_{N+1} \times \mathcal{V}_{N+1}$ , which includes both the leader and the followers. The pinning matrix  $\mathcal{P}$  is used to represent how each follower connects the leader, defined as  $\mathcal{P} = diag\{\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_N\}$ , where  $\mathcal{P}_i = 1$  if edge  $\{0, i\} \in \mathcal{E}_{N+1}$ ; otherwise  $\mathcal{P}_i = 0$ . Note that if  $\mathcal{P}_i = 1$ , then node *i* is said to be pinned to the leader. The leader accessible set of node *i* is defined as:

$$\mathbb{P}_i = \begin{cases} \{0\}, & \text{ if } \mathcal{P}_i = 1\\ \emptyset, & \text{ if } \mathcal{P}_i = 0 \end{cases}$$

A sequence of edges  $(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)$  with  $(i_{j-1}, i_j) \in \mathcal{E}_{N+1}$  for all  $j \in \{2, \dots, k\}$  is called a directed path from node  $i_1$  to  $i_k$ . A spanning tree is a directed path connecting all the nodes in the graph [95]. It is assumed that the augmented graph  $\mathcal{G}_{N+1}$  contains at least one spanning tree rooted at the leader for controllability [96]. In other words, the leader is globally reachable in  $\mathcal{G}_{N+1}$ , and every follower can obtain the leader information directly or indirectly. We can see that all the IFTs shown in Fig. 2 contain at least a spanning tree.

#### C. Model of Formation Geometry

The objective of platoon control is to track the speed of the leader and to maintain a desired formation governed by the inter-vehicle spacing policy,

$$\begin{cases} \lim_{t \to \infty} \|v_i(t) - v_0(t)\| = 0\\ \lim_{t \to \infty} \|p_{i-1}(t) - p_i(t) - d_{i-1,i}\| = 0 \end{cases}, i \in \mathcal{N},$$
(7)

where  $d_{i-1,i}$  is the desired space between node i-1 and node i. The selection of  $d_{i-1,i}$  determines the formation geometry of a vehicular platoon. For the CD policy,  $d_{i-1,i}$  is a given constant number,

$$d_{i-1,i} = d_0, \qquad i \in \mathcal{N},\tag{8}$$

where  $d_0$  is a positive constant number. For the CTH policy,  $d_{i-1,i}$  depends linearly on the velocity of ego-vehicle,

$$d_{i-1,i} = t_h v_i + d_0, \quad i \in \mathcal{N}, \tag{9}$$

where  $t_h$  is the time headway. For the ND policy,  $d_{i-1,i}$  is a nonlinear function of vehicle velocity, *i.e.*,

$$d_{i-1,i} = g(v_i), \qquad i \in \mathcal{N}. \tag{10}$$

#### D. Design of Distributed Controllers

The local controller in node *i* only uses its neighborhood information specified by  $\mathbb{I}_i = \mathbb{N}_i \cup \mathbb{P}_i$ . For the controller design methods, like robust control, SMC and MPC, there are specific procedures to use the neighborhood information; see [115] for basic descriptions of their applications in platoon control. For simplicity, here we only demonstrate the formulation of linear controllers,

$$\mu_{i}(t) = -\sum_{j \in \mathbb{I}_{i}} \left[ k_{ij,p} (p_{i}(t - \gamma_{ii}) - p_{j}(t - \gamma_{ij}) - d_{i,j}) + k_{ij,v} (v_{i}(t - \gamma_{ii}) - v_{j}(t - \gamma_{ij})) + k_{ij,a} (a_{i}(t - \gamma_{ij}) - a_{j}(t - \gamma_{ij})) \right],$$
(11)

where  $k_{ij,\#}$  (# = p, v, a) is the local controller gain,  $\gamma_{ii}$  is the time delay corresponding to obtain its own state, and  $\gamma_{ij}$  is the time delay corresponding to receive the state of node *j* through a communication channel. Note that many previous studies only employed specific types of (11), *e.g.*, [13][16][30][31].

# V. CASE STUDY: HOMOGENEOUS DYNAMICS, IDENTICAL CONTROLLER, AND RIGID FORMATION

This section studies a special case of platoon control under the four-component framework, *i.e.*, homogeneous dynamics, identical controllers, uniform time-delays and rigid formation. The 3<sup>rd</sup>-order state space model is adopted to describe the ND, as shown in (4). The platoon is assumed to be homogeneous (*i.e.*,  $A_i = A, i \in \mathcal{N}$ ) and the CD policy is employed. The DC (11) is identical, *i.e.*,  $k_{ij,\#} = k_{\#}$  (# = p, v, a),  $i, j \in \mathcal{N}$ . Besides, the time-delays are identical in all communication channels, *i.e.*,  $\gamma_{ij} = \gamma \ge 0, i, j \in \mathcal{N}$ . Such assumptions can be found in many previous studies, *e.g.*, [30][31][35][36].

The DC synthesis method is first discussed for the platoon without any time-delay. When the communication is perfect, *i.e.*,  $\gamma = 0$ , stable DC can be designed by solving an algebraic Riccati equation, in which the requirement of internal stability is enforced by limiting the minimum eigenvalue of  $\mathcal{L} + \mathcal{P}$ . Then, this technique is extended to the platoon with uniform time-delays, *i.e.*,  $\gamma > 0$ , where both the minimum eigenvalue of  $\mathcal{L} + \mathcal{P}$  and time-delay  $\gamma$  are conditioned.

This section only aims to present two application cases to illustrate the proposed four-component framework, which clearly demonstrates that all the four components (*i.e.*, ND, IFT, FG and DC) play an important role in the collective behavior of a platoon. Also, we limit our focus on platoons with linear controller and linear dynamics for the convenience of illustration and theoretical completeness. The analysis of nonlinear vehicle dynamics and more advanced control strategies is beyond the scope and the purpose of this paper.

#### A. Closed-loop Dynamics of Platoons

To rewrite (11) into a compact form, we first define a new tracking error  $\tilde{x}_i(t)$  for node *i* 

$$\tilde{x}_i(t) = x_i(t) - x_0(t) - \dot{d}_i$$
 (12)

where  $\tilde{d}_i = [d_{i,0}, 0, 0]^T$ . Note that under the CD policy, the desired inter-vehicle distance between *i*-th follower and the leader is given by  $d_{i,0} = -i \times d_0$ . Using identical controller gains and uniform time-delays, Eq. (11) is rewritten into

$$u_i(t) = -\sum_{j \in \mathbb{I}_i} k^T \left( \tilde{x}_i(t-\gamma) - \tilde{x}_j(t-\gamma) \right), \tag{13}$$

where  $k = [k_p, k_v, k_a]^T \in \mathbb{R}^{3 \times 1}$  is the vector of local control gains. To derive the collective dynamics of overall platoon, we define the collective state vector  $X = [\tilde{x}_1^T, \tilde{x}_2^T, \dots, \tilde{x}_N^T]^T \in \mathbb{R}^{3N \times 1}$  and the collective control input vector  $U = [u_1, u_2, \dots, u_N]^T \in \mathbb{R}^{N \times 1}$  for the followers. The collective dynamics of nodes from 1 to *N* are

$$\dot{X}(t) = I_N \otimes A \cdot X(t) + I_N \otimes B \cdot U(t)$$
(14)

with  $I_N \otimes A \in \mathbb{R}^{3N \times 3N}$ ,  $I_N \otimes B \in \mathbb{R}^{3N \times N}$ , where  $\otimes$  denotes the Kronecker product. According to (13), the collective distributed control law is written into:

$$U(t) = -(\mathcal{L} + \mathcal{P}) \otimes k^T \cdot X(t - \gamma).$$
(15)

Substituting (15) into (14), the closed-loop dynamics of the homogeneous platoon becomes:

$$\dot{X}(t) = I_N \otimes A \cdot X(t) - (\mathcal{L} + \mathcal{P}) \otimes Bk^T \cdot X(t - \gamma).$$
(16)

The following assumption on IFTs is used throughout this section.

Assumption 1: The graph  $\mathcal{G}_{N+1}$  contains a spanning tree rooted at the leader, and satisfies one of the following conditions:

- 1) Followers in a platoon are of "look-ahead" type, *i.e.*,  $\mathbb{N}_i = \{i h, \dots, i 1, \} \cap \mathcal{N}$ , where *h* is the number of nodes that node *i* can reach.
- Followers in a platoon are of symmetric "look-ahead & look-back" type, *i.e.*, N<sub>i</sub> = {i − h, ..., i + h} ∩ N \ {i}.
- Information flow among followers is undirected, *i.e.*, *j* ∈ N<sub>i</sub> ⇔ *i* ∈ N<sub>j</sub>, *i*, *j* ∈ N.

Note that **Assumption 1** defines a large class of IFTs, including all the topologies shown in Fig. 2.

*Lemma 1.* [24] Let  $\lambda_i, i \in \mathcal{N}$  denote the eigenvalues of matrix  $\mathcal{L} + \mathcal{P}$  corresponding to graph  $\mathcal{G}_{N+1}$ . If graph  $\mathcal{G}_{N+1}$  satisfies **Assumption 1**, then we have  $\lambda_i > 0$ ,  $i \in \mathcal{N}$ .

The stabilizing region for linear controller gains was explicitly derived in [24][32] for a homogenous platoon with perfect communications under **Assumption 1.** In this section, we present a synthesis method to design stable distributed controller for homogeneous platoons with/without time-delays.

**Remark 1.** As shown in (16), the closed-loop dynamics of a vehicular platoon is a function of four components, namely 1) ND, denoted by A, B; 2) IFT, denoted by  $\mathcal{L} + \mathcal{P}$ ; 3) FG, included in X as the desired distance (see (12)); and 4) DC, denoted by k. In addition, from (16), the performance measures of platoons must have a tight relationship with the four main components. However, it is usually rather difficult to explicitly obtain the relationship between performance metrics and decomposed components. Many research on string stability only focused on specific cases; see [5][8][11]-[14][17][29]. Besides, the scaling laws of stability margin were explicitly derived for specific IFTs in [32]-[36].

#### B. Controller Design without Communication Delay

Here, it is assumed that there are no time-delays, *i.e.*,  $\gamma = 0$ . Then, the closed-loop dynamics (16) become:

$$\dot{X}(t) = A_c \cdot X(t), \tag{17}$$

where  $A_C = I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes Bk^T$  is the overall closed-loop system matrix. The following theorem presents a straightforward method to design a stabilizing distributed controller for homogeneous platoons satisfying **Assumption 1**. *Theorem 1*. Consider a homogeneous platoon without time-delays with the closed-loop dynamics as (17). The distributed local controller gain k is chosen as

$$k^T = \alpha B^T P_{\varepsilon}, \tag{18}$$

where  $\alpha$  is a scaling factor, and  $P_{\varepsilon}$  is the unique positive definite solution to the algebraic Riccati equation (ARE)

$$A^{T}P_{\varepsilon} + P_{\varepsilon}A - P_{\varepsilon}BB^{T}P_{\varepsilon} + \varepsilon I_{3} = 0, \qquad (19)$$

where  $\varepsilon$  is a positive real number, called low-gain factor. Under **Assumption 1**, the platoon dynamics (17) are asymptotically stable if

$$\alpha \ge \frac{1}{2\min_{i \in \mathcal{N}} (\lambda_i)}.$$
(20)

**Proof:** The proof proceeds in two steps.

*Step 1.* We first prove that (17) is asymptotically stable as long as  $A - \lambda_i Bk^T$ ,  $\forall i \in \mathcal{N}$  are all Hurwitz.

From **Lemma 1**, we know that  $\lambda_i > 0$ ,  $i \in \mathcal{N}$ . There exists a nonsingular matrix W such that

$$W^{-1}(\mathcal{L} + \mathcal{P})W = J \tag{21}$$

where  $J \in \mathbb{R}^{N \times N}$  is the Jordan normal form of  $\mathcal{L} + \mathcal{P}$ , *i.e.*,

$$J = \begin{bmatrix} J_{n_1}(\lambda_1) & & \\ & J_{n_2}(\bar{\lambda}_2) & \\ & & \ddots & \\ & & & J_{n_r}(\bar{\lambda}_r) \end{bmatrix}$$

where  $\bar{\lambda}_i$  is the eigenvalue of  $\mathcal{L} + \mathcal{P}$ ,  $\sum_{i=1}^r n_i = N$  and  $J_{n_1}(\bar{\lambda}_1), J_{n_2}(\bar{\lambda}_2), \dots, J_{n_r}(\bar{\lambda}_r)$  are Jordan blocks of sizes  $n_1, n_2, \dots, n_r$ . Then, a similarity transformation of  $A_c$  gives a block triangular matrix, *i.e.*,

$$\tilde{A}_{c} = (W \otimes I_{N})^{-1} \cdot A_{c} \cdot (W \otimes I_{N})$$

$$= (W \otimes I_{N})^{-1} \cdot (I_{N} \otimes A - (\mathcal{L} + \mathcal{P}) \otimes Bk^{T})$$

$$\cdot (W \otimes I_{N})$$
(22)

$$= I_N \otimes A - J \otimes Bk^T.$$

Denote the diagonal entries of *J* as  $\{\lambda_1, \lambda_2, \dots, \lambda_N\}$ , *i.e.*,

$$\{\lambda_1, \lambda_2, \cdots, \lambda_N\} = \left\{ \underbrace{\bar{\lambda}_1, \cdots, \bar{\lambda}_1}_{n_1}, \cdots, \underbrace{\bar{\lambda}_r, \cdots, \bar{\lambda}_r}_{n_r} \right\}.$$

Then, we have

$$S(A_c) = S(\tilde{A}_c) = \bigcup_{i=1}^{N} \{S(A - \lambda_i B k^T)\},$$
(23)

where S(A) is the spectrum of matrix A. Thus, the platoon dynamics (17) is asymptotically stable if and only if  $A - \lambda_i Bk^T$ ,  $i \in \mathcal{N}$  are all Hurwitz.

Step 2. We prove that  $A - \lambda_i Bk^T$ ,  $i \in \mathcal{N}$  are all Hurwitz if (20) is satisfied.

Let  $P_{\varepsilon}$  be a solution to ARE (19). Then, we know

$$(A - \lambda_i B k^T)^T P_{\varepsilon} + P_{\varepsilon} (A - \lambda_i B k^T)$$
(24)

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$$= A^T P_{\varepsilon} + P_{\varepsilon} A - \lambda_i (k B^T P_{\varepsilon} + P_{\varepsilon} B k^T).$$

Based on (18), we have

$$\begin{cases} kB^T P_{\varepsilon} = \alpha P_{\varepsilon} BB^T P_{\varepsilon} \\ P_{\varepsilon} Bk^T = \alpha P_{\varepsilon} BB^T P_{\varepsilon} \end{cases}$$
(25)

Hence, combining (19) and (25), (24) becomes

$$(A - \lambda_i B k^T)^T P_{\varepsilon} + P_{\varepsilon} (A - \lambda_i B k^T)$$
  
=  $(1 - 2\lambda_i \alpha) P_{\varepsilon} B B^T P_{\varepsilon} - \varepsilon I_3$  (26)

Since  $P_{\varepsilon}BB^TP_{\varepsilon} > 0$  and  $\varepsilon I_3 > 0$ , by Lyapunov theory, we know matrix  $A - \lambda_i B k^T$  is Hurwitz if  $1 - 2\lambda_i \alpha \leq 0$ ,  $\forall i \in \mathcal{N}$ . Thus, if the parameter  $\alpha$  is chosen to satisfy condition (20),  $A - \lambda_i B k^T, \forall i \in \mathcal{N}$  are all Hurwitz.

Combining the results in steps 1 and 2, we complete the proof.

The similarity transformation in (22) decomposes a platoon (17) into multiple sub-systems, which can be easily handled. This kind of technique originated in the pioneering work by Fax et al. [63], which has been used by many researchers subsequently, e.g., [98][99][102]. The approach in Theorem 1 decouples the controller design from the IFT, and IFT only influences the choice of scaling factor  $\alpha$  by condition (20), which is called the least eigenvalue condition. Note that the computation complexity of ARE (19) is scalable and independent of platoon size. Similar techniques can be found in [100][101].

Remark 2. A systematic distributed overlapping control strategy was proposed in [17], which used the inclusion principle to decompose an interconnected platoon into locally decoupled subsystems. However, such method was only suitable for the PL and PLF topologies. Theorem 1 can cover a large class of IFTs, including all the topologies in Fig. 2, which provides a straightforward approach to design the controller gains to guarantee the internal stability. One main drawback of the method in **Theorem 1** is that the scaling factor  $\alpha$  will approach infinity if the least eigenvalue of  $\mathcal{L} + \mathcal{P}$  goes to zero as the platoon size increases. This phenomenon will happen for the BD topology and some undirected topologies, which can be viewed as fundamental limitations for such topologies [32][39].

#### C. Controller Design with Uniform Communication Delays

In this section, it is assumed that there exist uniform time-delays, *i.e.*,  $\gamma > 0$ . Before presenting the design process of stabilizing distributed controllers for platoons with uniform time-delays, we need the following Lemmas.

Lemma 2. [103] Considering a linear time-delay system

$$\dot{x} = Ax + A_d x (t - \gamma). \tag{27}$$

Assume  $A + A_d$  is Hurwitz. Then system (27) is asymptotically stable for  $\gamma \in [0, \gamma^*]$  if

$$\det[j\omega I - A - e^{-j\omega\gamma}A_d] \neq 0, \forall \omega \in \mathbb{R}, \forall \gamma \in [0, \gamma^*].$$
(28)

where  $det(\cdot)$  denotes the determinant.

Lemma 3. [104] Assume  $P_{\rm s}$  is the unique positive definite solution of the ARE (19). Then, we have

$$\lim_{\varepsilon \to 0} P_{\varepsilon} = 0. \tag{29}$$

*Lemma 4.* [97] Let  $Q \in \mathbb{C}^{n \times n}$ ,  $W \in \mathbb{C}^{n \times n}$  and the corresponding singular values be arranged in a non-increasing way, as  $\bar{\sigma}(\cdot) =$  $\sigma_1(\cdot) \ge \sigma_2(\cdot) \ge \cdots \ge \sigma_n(\cdot) = \underline{\sigma}(\cdot)$ . Then, we have

$$\prod_{i=1}^{n} \sigma_i(Q) = |\det(Q)|, \tag{30}$$

$$\sum_{i=1}^{n} \sigma_i^2(Q) = \|Q\|_F^2, \tag{31}$$

$$\underline{\sigma}(Q-W) \ge \underline{\sigma}(Q) - \overline{\sigma}(W), \tag{32}$$

where  $\|\cdot\|$  represents the modulus, and  $\|\cdot\|_F$  is the Frobenius norm of a matrix.

Theorem 2. Consider a homogeneous platoon under uniform time-delays with the closed-loop dynamics as (16). The distributed local controller gain k is chosen as (18). Choose the scaling factor  $\alpha$ , such that the following condition is satisfied:

$$\alpha \ge \frac{1}{\min_{i \in \mathcal{N}} (\lambda_i)}.$$
(33)

Then, under Assumption 1, for any given  $\gamma^* \ge 0$ , there exists  $\varepsilon^* > 0$  such that platoon dynamics (16) are asymptotically stable for any low-gain factor  $\varepsilon \in (0, \varepsilon^*]$ and  $\gamma \in (0, \gamma^*]$ .

**Proof:** the proof of **Theorem 2** also proceeds in two steps.

Step 1. We first prove that the platoon dynamics (16) are asymptotically stable if and only if the following N subsystems

$$\dot{\xi}_i(t) = A\xi_i(t) + \lambda_i Bk^T \cdot \xi_i(t-\gamma), \forall i \in \mathcal{N},$$
(34)

are all asymptotically stable. To see this, we know that the platoon dynamics (16) can be represented as:

$$\dot{X}(t) = (W \otimes I_N)(I_N \otimes A)(W^{-1} \otimes I_N) \cdot X(t) -(W \otimes I_N)(J \otimes Bk^T)(W^{-1} \otimes I_N) \cdot X(t-\gamma),$$
(35)

where we used the similarity transformation (21). Further, the above equation can be rewritten as

$$\dot{\chi}(t) = (I_N \otimes A) \cdot \chi(t) - (J \otimes Bk^T) \cdot \chi(t - \gamma),$$
(36)  
n which,  $\chi = [\xi_1^T, \xi_2^T, \dots, \xi_N^T] = (W^{-1} \otimes I_N) \cdot X.$ 

If  $\lim_{t \to \infty} |\xi_i(t)| = 0, \forall i \in \mathcal{N}$ , then it follows from  $X = (W \otimes I_N) \cdot \chi$  that  $\lim_{t \to \infty} |x_i(t)| = 0, \forall i \in \mathcal{N}$ , which means the internal stability is achieved. Based on (21) and (36), the dynamics of  $\xi_i(t)$  obey the following equations:

$$\dot{\xi}_{i}(t) = A\xi_{i}(t) + \lambda_{i}Bk^{T} \cdot \xi_{i}(t-\gamma) + \delta_{i}Bk^{T} \\ \cdot \xi_{i+1}(t-\gamma), \forall i \in \mathcal{N},$$
(37)

where  $\delta_i$  equals to 1 or 0,  $\forall i \in \mathcal{N}$ ; and  $\xi_{N+1}(t) \equiv 0$ . Clearly, the stability of (37) is equivalent to the stability of (34). Thus, the platoon dynamics (16) are asymptotically stable if and only if the N subsystems in (34) are all asymptotically stable.

Step 2. Based on **Theorem 1**, we have  $A - \lambda_i Bk^T$ ,  $\forall i \in \mathcal{N}$ is Hurwitz, if condition (33) is satisfied. Then, according to **Lemma 2**, we only need to prove for any given  $\gamma^* \ge 0$ , there exists  $\varepsilon^* > 0$ , such that for any  $\varepsilon \in (0, \varepsilon^*]$  and  $\gamma \in (0, \gamma^*]$ , the following condition is satisfied.

$$det[j\omega I - A + e^{-j\omega\gamma}\lambda_i Bk^T] = det[j\omega I - A + \alpha\lambda_i e^{-j\omega\gamma} BB^T P_{\varepsilon}] \neq 0,$$

$$\forall i \in \mathcal{N}, \forall \omega \in \mathbb{R}, \forall \gamma \in [0, \gamma^*].$$
(38)

Clearly, for any given  $\gamma^* \ge 0$ , there exists  $\delta > 0$ , such that

$$\cos(\omega\gamma) > 1/2, s.t. |\omega| < \delta, \gamma \in (0, \gamma^*].$$
<sup>(39)</sup>

Next, we split the proof of (38) into two cases in which  $|\omega| < \delta$  and  $|\omega| \ge \delta$ , respectively.

**Case 1.** If  $|\omega| < \delta$ , based on (39) and (33), we know

$$\alpha\lambda_{i}\cos(\omega\gamma) > \frac{1}{2}, \gamma \in (0,\gamma^{*}], \forall i \in \mathcal{N}, \qquad (40)$$

Considering (40) and ARE (19), we have

$$[A - \alpha \lambda_i e^{-j\omega\gamma} B B^T P_{\varepsilon}]^T P_{\varepsilon} + P_{\varepsilon} [A - \alpha \lambda_i e^{-j\omega\gamma} B B^T P_{\varepsilon}]$$
(41)  
=  $(1 - 2\lambda_i \alpha \cos(\omega\gamma)) P_{\varepsilon} B B^T P_{\varepsilon} - \varepsilon I_{\varepsilon} < 0.$ 

Thus,  $A - \alpha \lambda_i e^{-j\omega \gamma} B B^T P_{\varepsilon}$  is Hurwitz, and (38) holds with  $|\omega| < \delta$ .

**Case 2.** It remains to verify condition (38) with  $|\omega| \ge \delta$ . According to the definition of state matrix A in (4) and Lemma 4, we have

$$|\det(j\omega I - A)| = \omega^2 \sqrt{\left(\omega^2 + \frac{1}{\tau^2}\right)} = \prod_{i=1}^3 \sigma_i (j\omega I - A), \quad (42)$$
  
and

$$\|j\omega I - A\|_F^2 = 3\omega^2 + 2 + \frac{1}{\tau^2} = \sum_{i=1}^3 \sigma_i^2 (j\omega I - A).$$
(43)

Based on (42) and (43), we know

$$\underline{\sigma}(j\omega I - A) \ge \frac{\omega^2 \sqrt{\left(\omega^2 + \frac{1}{\tau^2}\right)}}{3\omega^2 + 2 + \frac{1}{\tau^2}} = \frac{\sqrt{\left(\omega^2 + \frac{1}{\tau^2}\right)}}{3 + \frac{2}{\omega^2} + \frac{1}{\omega^2\tau^2}}.$$
 (44)

Therefore, there exists  $\mu_0 > 0$ , such that

$$\underline{\sigma}(j\omega I - A) \ge \mu_0, \ s.t. |\omega| \ge \delta.$$
(45)

Given  $\alpha$  and  $\gamma \in (0, \gamma^*]$ , and based on **Lemma 3**, we have

$$\lim_{\varepsilon \to 0} (\alpha \lambda_i e^{-j\omega \gamma} P_{\varepsilon}) = 0, \quad \forall i \in \mathcal{N}.$$
(46)

Hence, there exists  $\varepsilon^* > 0$  such that, when  $\varepsilon \in (0, \varepsilon^*]$ , we have

$$\bar{\sigma}(\alpha\lambda_i e^{-j\omega\gamma}P_{\varepsilon}) < \frac{\mu_0}{2}, \quad \forall i \in \mathcal{N}.$$
(47)

Combing (44) and (47), we know

$$\underline{\sigma}(j\omega I - A - \alpha\lambda_i e^{-j\omega\gamma} P_{\varepsilon}) \ge \mu_0 - \frac{\mu_0}{2} > 0,$$
  
$$\forall i \in \mathcal{N}, \varepsilon \in (0, \varepsilon^*], |\omega| \ge \delta.$$
(48)

Therefore, for any given  $\gamma^* \ge 0$ , there exists  $\varepsilon^* > 0$ , such that for any  $\varepsilon \in (0, \varepsilon^*]$  and  $\gamma \in (0, \gamma^*]$ , condition (38) holds with  $|\omega| \geq \delta$ . Under these conditions, platoon dynamics (16) are asymptotically stable. ■

To guarantee the internal stability for platoons without time-delays, only the scaling factor  $\alpha$  needs to satisfy condition (20), while for platoons with time-delays, the low-gain factor  $\varepsilon$  also needs to be sufficiently small, which was called the low gain approach in [107].

*Remark* 3. There are four parameters, *i.e.*,  $\alpha$ ,  $\delta$ ,  $\mu_0$ ,  $\varepsilon^*$ , that need to be chosen sequentially in the process of controller synthesis. First, the scaling factor  $\alpha$  is selected by condition (20) according to the properties of the given IFT. Then,  $\delta$  is chosen based on the time-delay value  $\gamma^*$  and the choice of scaling factor  $\alpha$ . Such a  $\delta$  yields the value of  $\mu_0$ . Eventually,  $\varepsilon^*$ is determined by  $\mu_0, \alpha, \gamma$ .

#### D. Numerical Simulations

Here, numerical simulations are conducted to illustrate the effectiveness of the distributed controller synthesis methods in Theorems 1 and 2. We consider a homogeneous platoon with 11 identical vehicles (1 leader and 10 followers) interconnected by the six IFTs shown in Fig. 2. The acceleration or deceleration of the leader can be viewed as disturbances in a

platoon. The initial state of the leader is set as  $p_0(t) = 0$ ,  $v_0 =$ 20 m/s, and the desired trajectory is given by

$$v_0 = \begin{cases} 20 \text{ m/s} & t \le 5 \text{ s} \\ 20 + 2t \text{ m/s} & 5 \text{ s} < t \le 10 \text{ s} \\ 30 \text{ m/s} & t > 10 \text{ s} \end{cases}$$

In the simulations, the desired spacing is set as  $d_{i-1,i} =$ 20 m and the vehicle length is equal to 4 m. The inertial delay of vehicle longitudinal dynamics is all set as  $\tau_i = 0.5$  s,  $i \in \mathcal{N}$ . The initial state of the platoon is set as the desired state, *i.e.*, the initial spacing errors and velocity errors are all equal to 0.



Fig. 3 Spacing errors for platoons without communication delays under different IFTs. (a): PF; (b): PLF; (c):BD; (d):BDL; (e): TPF; (f):TPLF

For the case where there are no time delays, *i.e.*,  $\gamma = 0$ , the scaling factor  $\alpha$  should satisfy the condition (20), and another low-gain factor  $\varepsilon$  can be any positive real number according to Theorem 1. TABLE II. shows the chosen parameters, which obviously satisfy the requirements of Theorem 1. As shown in Fig. 3, the motions of the vehicles are stable for platoons under

all IFTs, which confirms the results of **Theorem 1**. It is noted that for the spacing errors under the PLF, BDL and TPLF topologies, only the first follower has non-zero spacing error and the other followers have near-zero spacing errors. The reason is that all the followers in the platoon are pinned to the leader and have zero initial errors. Hence, the followers have similar dynamic evolution under the linear identical controller (15), which means the spacing errors between adjacent followers are all close to zero.

For the case where there exist uniform time delays, *i.e.*,  $\gamma > 0$  in the communication channels, it is assumed that  $\gamma = 0.5$  s for simulations. According to **Theorem 2**, four parameters, *i.e.*,  $\alpha$ ,  $\delta$ ,  $\mu_0$ ,  $\varepsilon^*$ , need to be chosen sequentially in the process of controller synthesis, which are shown in TABLE III. As shown in Fig. 3, the motions of the vehicles are stable for platoons with time-delays under all IFTs, which obviously agree with the results of **Theorem 2**.

TABLE II. DESIGN PARAMETERS WIHTOUT COMMUNICATION DELAY

Topology	$\lambda_{min}$	α	Е
PF	1	0.5	1
PLF	1	0.5	1
BD	0.022	22.5	1
BDL	1	0.5	1
TPF	1	0.5	1
TPLF	1	0.5	1

Notation:  $\lambda_{min} = \min_{i \in \mathcal{M}} \{\lambda_i\}$ 

TABLE III. DESIGN PARAMETERS WITH COMMUNICATION DELAY

Topology	$\lambda_{min}$	$\lambda_{max}$	α	δ	$\mu_0$	Е
PF	1	1	1	2	0.63	$6.1 \times 10^{-4}$
PLF	1	2	1	2	0.63	$3.8 \times 10^{-5}$
BD	0.022	3.91	45	2	0.63	$5.7 \times 10^{-13}$
BDL	1	4.90	1	2	0.63	$1.2 \times 10^{-6}$
TPF	1	2	1	2	0.63	$3.8 \times 10^{-5}$
TPLF	1	3	1	2	0.63	9.5×10 <sup>-6</sup>

Notation: 
$$\lambda_{max} = \max_{i=1}^{N} \{\lambda_i\}$$





Fig. 4 Spacing errors for platoons with uniform communication delays under different IFTs. (a): PF; (b): PLF; (c):BD; (d):BDL; (e): TPF; (f):TPLF

#### VI. CONCLUSION

This paper introduced a four-component based framework for platoon control systems from the perspective of networked control. The networked control perspective allows us to decompose a platoon into four interrelated components, *i.e.*, 1) node dynamics, 2) information flow topology, 3) formation geometry, and 4) distributed controller. Then, the unified mathematical models were introduced for homogeneous platoons with linear dynamics and controllers. To demonstrate the effectiveness of this framework, a distributed controller synthesis approach was designed by converting the platoon control problem to a parametric algebraic Riccati equation. The designed controllers can guarantee the internal stability for a variety of topologies in the presence of uniform time-delays.

Although some theoretical results have been provided in the literature, there are still some open questions, especially considering the emerging large-scale applications of V2V communications. Three topics are briefly discussed here:

1) How to model, analyze, and design platoon control in a systematic way? The future challenges come from the nonlinearity of node dynamics, the diversity of topologies, and the low-cost demands of distributed controllers. Communication issues such as data delay, quantization error, and packet loss also significantly challenge platoon control. Other interesting questions include how to optimize the design of topologies and controllers when considering both the platoon performances and communication issues.

2) How to balance the various performance metrics in a platoon system, especially considering practical requirements from traffic systems? The balance of string stability, stability

margin, and coherence behavior is attracting increasing interest now. Moreover, the ultimate goals of platooning are to enhance highway safety, improve traffic utility, and reduce fuel consumption. How to consider these practical performance requirements is rather challenging for platoon control.

3) How to develop scalable methods for large-scale heterogeneous platoons? For homogeneous platoons, similarity transformation and matrix factorization have been widely used to decompose a big platoon into multiple subsystems [32][39] [63]. However, this technique is not suitable for heterogeneous platoons (either heterogeneous feedbacks or heterogeneous dynamics). For this direction, we note that chordal decomposition might be a promising technique to deal with heterogeneous issue; see a recent application on structured controller design[106].

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