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AUTOMATED MODELING AND MODE SCREENING FOR EXHAUSTIVE SEARCH OF DOUBLE-PLANETARY-GEAR POWER SPLIT HYBRID POWERTRAINS

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ABSTRACT

Double Planetary Gear (PG) power-split hybrid powertrains have been used in production vehicles from Toyota and General Motors. Some of the designs use clutches to achieve multiple operating modes to improve powertrain operation flexibility and efficiency at the expense of higher complexity. In this paper, an automatic modeling and screening process is developed, which enables exhaustively search through all designs with different configurations, clutch locations and operating modes. A case study was conducted based on the configuration used in the model year 2010 Prius and Camry hybrids. It was found that by adding clutches, fuel economy can be improved significantly for plug-in hybrid (charge depletion) operations.

NOMENCLATURE

ω	Rotational speed of powertrain component
$\dot{\omega}$	Rotational acceleration of powertrain component
$\dot{\Omega}$	Generalized acceleration
A	Dynamic matrix
I	Inertia of powertrain component
J	Inertia matrix
F	Internal force between gears on the planetary gearset
K	Final drive gear ratio on output shaft
M	transformation matrix
R	Gear radius of the ring gear
S	Gear radius of the sun gear
T	Torque of powertrain component
e (subscript)	Engine
MG (subscript)	Electric machine
out (subscript)	Output shaft

INTRODUCTION

The hybrid electric powertrain is one of the most important technologies to meet the challenging fuel economy standards set by the EU and US governments [1]. Hybrid and electric car sales increased by 73% in 2012 in the U.S. 473,000 hybrids and plug-in hybrids were sold, which is 3.3% of the market, a significant increase from 2.2% market share in 2011[2].

90% of the strong hybrid vehicle sales are power-split type [3], which utilizes one or more planetary gears as the transmission device. Toyota Prius, Ford Fusion and Chevrolet Volt are all power-split hybrid vehicles. The planetary gears are compact, high-capacity and very efficient. In addition, they perform as an Electronic Continuous Variable Transmission (ECVT) when the electric machines are properly controlled. When the powertrain devices are sized and controlled well, the hybrid vehicle can achieve good drivability and excellent fuel economy simultaneously.

When clutches are added, different operating modes can be used, which add flexibility to vehicle operation. For example, an input-spit mode can be used for better launching performance while a compound-split mode can be used for better high-speed driving while curtailing the operating speed of the electric machines [4]. It is also possible to have parallel modes, series modes, pure EV modes and fixed-gear modes on the same powertrain [5][6][7]. Having a number of operating modes makes it possible to fully realize the potential of the powertrain.

Although many configurations and designs have been patented and some implemented commercially, much more remain unexplored. “Configuration” in this paper refers to the way that the power devices (engine and generator/motors) and output shaft are connected to the nodes of PGs. An exhaustive analysis of all possible configurations of power-split vehicles using a single PG was reported in [8]. For power-split vehicles using more than one PG, a general modeling method was also developed [9]. However, general clutch allocation, mode screening and identification of unique modes have not been discussed in the literature. In this paper, an automated modeling methodology will be proposed, which covers models including all possible clutch locations to generate all possible modes. A systematic mode identification method is carried out, with only feasible and unique modes kept for design and control study.

Once a particular configuration is selected and all its feasible modes identified (as an example, in this paper, we will study the Prius 2010 configuration), our methodology will answer the following questions: how many clutches can be added and how many distinct modes can we have? Among all possible modes, how many of them are useful? How much benefit can we get? Where do those benefits come from?

The paper is organized as follows: In Section II, we will illustrate the dynamics of the Planetary Gear (PG) system and present an automated modeling procedure as well as the mode screening and identification process for double PG systems. In Section III, the configuration of THS-II is chosen for a case study, a detailed analysis of multiple mode operation for a specific design is presented. And finally, in Section IV, the conclusions are presented.

DYNAMICS OF PLANETARY GEAR AND AUTOMATED MODELING

A planetary gear (PG) system consists of a ring gear, a sun gear, and a carrier with several pinion gears. A lever analogy [12] can be used to represent the 2 degree of freedom (DoF) dynamics of this single planetary gear, as shown in Figure 1. The rotational speeds and acceleration of the three nodes (sun gear, ring gear, carrier) must satisfy the constraint shown in Eq. (1), where the subscripts s, r, c indicate the sun gear, ring gear and the carrier. *S* and *R* are the radii of the sun gear and ring gear, respectively.

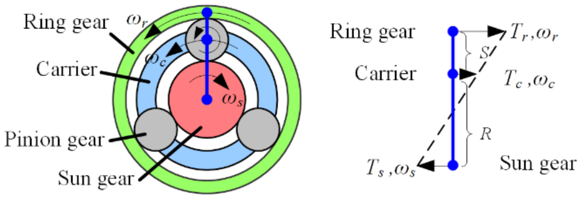

Figure 1. Planetary Gear and its Lever Analogy

$$\omega_s S + \omega_r R = \omega_c \left(R + S\right) \tag{1}$$

The dynamics of PG system can be represented using state-space form as suggested in the literature [9]. In this paper, a more general form of modeling will be presented, with all possible clutch locations and modes considered.

2.1 Double Planetary Gear System

Many of today’s power-split hybrid vehicles use two Motor/Generators (MGs) to complement the engine. In this research, we only consider the case that each planetary gear is connected with two powertrain components, since having three powertrain components on the same PG will lead to very limited design flexibility. Therefore, the number of configurations is $C_4^2 P_3^2 P_3^2 = 216$. In addition, they can be separated into two categories, depending on whether the engine and vehicles are on the same PG or not, as shown in Figure 2. Since varying the connection of node on one planetary gear will only change the relative speed ratio but not the function of the mode, for each category, they have the same number and types of modes. As an example configuration of type (a), Figure 3 shows the connection used in the Prius, Camry hybrid and Highlander hybrid since model year 2010.

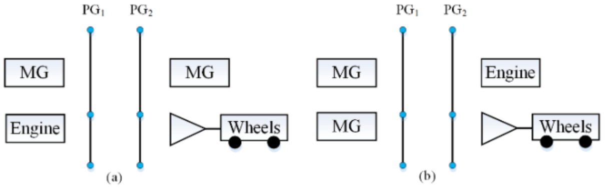

Figure 2. Two configuration types: engine and vehicle on the different PG (a) or same PG (b)

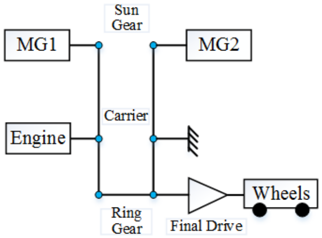

Figure 3. The lever diagram of Prius model year 2010 powertrain

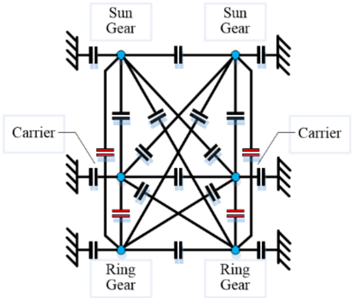

Figure 4. All 21 possible clutch locations for a double PG system, the four highlighted in red are redundant and the one that ground the vehicle node is considered infeasible in this paper

For a given double PG configuration, at most 21 clutches can be added, including 6 grounding clutches, and 15 (C_6^2) clutches between any two nodes, as shown in Figure 4. However, for each planetary gear, locking any two nodes makes all three nodes rotating at the same speed so really only one such clutch is needed for each PG—the redundant clutches are marked in red in Figure 4. In addition, the grounding clutch for the vehicle output shaft is meaningless during driving, so at the end “only” 16 clutch locations are studied.

While we will start by studying the design cases with all 16 clutches, it is clear the obtained results only serve as a benchmark and cannot be easily implemented in practice. In addition, it is questionable that we really need all the modes enabled by 16 clutches. In this paper, we consider the special case of adding 3 clutches in our case study. Since a double PG system initially has 4 DoF without any connection, and each non-redundant clutch engagement reduces system DoF by one, at most 3 clutches are needed to be engaged simultaneously. Moreover, it may lead to as many as 7 different modes which can lead to feasible and sub-optimal designs. In addition, the Chevy Volt uses 3 clutches, so it is feasible in practice. To avoid redundant designs and to facilitate this systematical automatic modeling procedure, an *assumption* is made: any node cannot be connected to all three nodes on the other PG at the same time since it is functionally the same when it is connected with 2 nodes on the other PG.

2.2 Automated Modeling

In this subsection, the automated modeling process for double-PG is described.

*<u>Step 1</u>: Initialize A_0 matrix_*

An 8x8 zero matrix is first created and it will be decomposed into four parts $\begin{bmatrix} J & D \\ D^T & 0 \end{bmatrix}$ where J is a 6x6 matrix. The first four elements of the principal diagonal are replaced by the inertias of the vehicle, engine, MG1 and MG2. The remaining two diagonal entries of the sub-matrix *J* will be filled with the planetary gear node which is not connected with any powertrain components, following the subsequent order: ring gear, carrier and sun gear, from the first PG to the second PG.

The connections of the planetary gear nodes with the 4 components determine the entries of the upper-right 6x2 sub-matrix *D* and the 2x6 sub-matrix D^T. The number of columns of *D* is equal to the number of PGs. When one powertrain component is connected to a PG node, the “node coefficient” will be entered in the *D* matrix: $-S_{(\cdot)}$ if it is connected to the sun gear; $-R_{(\cdot)}$ if it is connected to the ring gear; $R_{(\cdot)}+S_{(\cdot)}$ if it is connected to the carrier, where $S_{(\cdot)}$, $R_{(\cdot)}$ are the radii of the ring gear and the sun gear of the two PG. The dynamic relationship between the component torque T_0 and generalized acceleration vectors $\dot{\Omega}_0$ are shown in Eq.(2) where $\dot{\Omega}$ consists of components’ angular acceleration and F represents the internal force in PG system. The matrices for the configuration used in Prius 2010 are shown in Eq. (3).

$$A_0\dot{\Omega}_0 = \begin{bmatrix} J & D \\ D^T & 0 \end{bmatrix}\begin{bmatrix} \dot{\Omega} \\ F \end{bmatrix} = \begin{bmatrix} T \\ 0 \end{bmatrix} = T_0 \qquad (2)$$

$$A_0 = \begin{bmatrix} I_{out}+I_{r2} & 0 & 0 & 0 & 0 & 0 & 0 & -R_2 \\ 0 & I_e+I_{c1} & 0 & 0 & 0 & 0 & R_1+S_1 & 0 \\ 0 & 0 & I_{MG1}+I_{s1} & 0 & 0 & 0 & -S_1 & 0 \\ 0 & 0 & 0 & I_{MG2}+I_{s2} & 0 & 0 & 0 & -S_2 \\ 0 & 0 & 0 & 0 & I_{r1} & 0 & -R_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{c2} & 0 & R_2+S_2 \\ 0 & R_1+S_1 & -S_1 & 0 & -R_1 & 0 & 0 & 0 \\ -R_2 & 0 & 0 & -S_2 & 0 & R_2+S_2 & 0 & 0 \end{bmatrix}, \qquad (3)$$

$$T_0 = \begin{bmatrix} T_{Load} & T_e & T_{MG1} & T_{MG2} & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\dot{\Omega}_0 = \begin{bmatrix} \dot{\omega}_{out} & \dot{\omega}_{eng} & \dot{\omega}_{MG1} & \dot{\omega}_{MG2} & \dot{\omega}_{r1} & \dot{\omega}_{c2} & F_1 & F_2 \end{bmatrix}^T$$

<u>Step 2</u>: Define Transformation Matrix

Transformation matrices *M* and *P* are defined according to the clutch placement and engagement. *M* is initialized as an 8x8 identity matrix. When the i^{th} node is connected with the j^{th} node, without losing generality, assuming $i < j$, the processes shown in Eqs.(4) and (5) are used to update the *M* matrix. If the clutch is engaged to ground the i^{th} node, i^{th} row = [], which means the row is eliminated. After this step, *M* becomes an $(8-n)\times 8$ matrix where n is the number of clutches engaged and $1 \le n \le 3$.

The generation of *P* is similar to that of *M* but only row elimination (Eq. (5)) is applied. *P* is also an $(8-n)\times 8$ matrix.

$$i^{th}\,row = i^{th}\,row + j^{th}\,row \qquad (4)$$

$$j^{th}\,row = [\,] \qquad (5)$$

<u>Step 3</u>: Formulate the Dynamic Equation

The dynamic matrix *A* of the powertrain system with clutch engagement is generated through Eq.(6). The system dynamics of a certain mode can be according to Eq.(7). As an example, Eq.(8) and (9) show the equations of the Prius 2010 depicted in Figure 3.

$$A = MA_0M^T,\ T = MT_0,\ \dot{\Omega} = P\dot{\Omega}_0 \qquad (6)$$

$$A\dot{\Omega} = T \qquad (7)$$

$$M = \begin{bmatrix} 1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}, P = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix} \qquad (8)$$

2.3 Mode Screening

With multiple clutch operation, various modes can be achieved. When the vehicle cannot be powered by any powertrain component, it is defined as an infeasible mode. For modes having identical dynamic equations, only one mode is kept and the rest are denoted as redundant modes. In this section, the processes to identify and eliminate infeasible and redundant modes are described.

$$A=\begin{bmatrix} I_{out}+I_{r2}+I_{r1} & 0 & 0 & 0 & -R_1 & -R_2 \\ 0 & I_e+I_{c1} & 0 & 0 & R_1+R_2 & 0 \\ 0 & 0 & I_{MG1}+I_{s1} & 0 & -S_1 & 0 \\ 0 & 0 & 0 & I_{MG2}+I_{s2} & 0 & -S_2 \\ -R_1 & R_1+R_2 & -S_1 & 0 & 0 & 0 \\ -R_2 & 0 & 0 & -S_2 & 0 & 0 \end{bmatrix} \tag{9}$$

$$T=\begin{bmatrix} T_{Load} \\ T_e \\ T_{MG1} \\ T_{MG2} \\ 0 \\ 0 \end{bmatrix}, \dot{\Omega}=\begin{bmatrix} \dot{\omega}_{out} \\ \dot{\omega}_e \\ \dot{\omega}_{MG1} \\ \dot{\omega}_{MG2} \\ F_1 \\ F_2 \end{bmatrix}$$

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<u>*Step 1*</u>*: Construct A^* matrix*

The *A* matrix is inverted to obtain the dynamic equation that relates input to state derivatives. With the previously defined process, the *A* matrix is always invertible. Meanwhile, not every element of the A^{-1} matrix is useful. The useful part of A^{-1} is extracted, to obtain a final 4×4 matrix A^*, as shown in Eq.(10).

$$\begin{bmatrix} \dot{\omega}_{out} \\ \dot{\omega}_{eng} \\ \dot{\omega}_{mg1} \\ \dot{\omega}_{mg2} \end{bmatrix} = A^* \begin{bmatrix} T_{load} \\ T_{eng} \\ T_{mg1} \\ T_{mg2} \end{bmatrix} \tag{10}$$

In order to construct the A^* matrix, the last two columns and rows as well as the column and rows associated with any free node (node with no powertrain component attached) in A^{-1} are eliminated because they have no impact to the final state equation. There are two cases after the elimination:

(1) If there is no powertrain components collocation due to clutch engagement, the A^* matrix is acquired after the elimination process described above. For the Prius 2010 powertrain, its A^{-1} and A^* are shown in Eq. (11).

(2)If there is collocation, the torque coefficients corresponding to the collocated components are duplicated to make the sequence of the coefficients in A^* correspond to "output", "engine", "MG1" and "MG2", which will lead to identical rows in the A^* matrix. An example parallel mode and its corresponding A^{-1} and A^* matrices are shown in Figure 5 and Eq.(12).

$$A^{-1}=\begin{bmatrix} A_{11}^{inv} & A_{12}^{inv} & A_{13}^{inv} & A_{14}^{inv} & A_{15}^{inv} & A_{16}^{inv} \\ A_{21}^{inv} & A_{22}^{inv} & A_{23}^{inv} & A_{24}^{inv} & A_{25}^{inv} & A_{26}^{inv} \\ A_{31}^{inv} & A_{32}^{inv} & A_{33}^{inv} & A_{34}^{inv} & A_{35}^{inv} & A_{36}^{inv} \\ A_{41}^{inv} & A_{42}^{inv} & A_{43}^{inv} & A_{44}^{inv} & A_{45}^{inv} & A_{46}^{inv} \\ A_{51}^{inv} & A_{52}^{inv} & A_{53}^{inv} & A_{54}^{inv} & A_{55}^{inv} & A_{56}^{inv} \\ A_{61}^{inv} & A_{62}^{inv} & A_{63}^{inv} & A_{64}^{inv} & A_{65}^{inv} & A_{66}^{inv} \end{bmatrix}$$

$$A^*=\begin{bmatrix} A_{11}^{inv} & A_{12}^{inv} & A_{13}^{inv} & A_{14}^{inv} \\ A_{21}^{inv} & A_{22}^{inv} & A_{23}^{inv} & A_{24}^{inv} \\ A_{31}^{inv} & A_{32}^{inv} & A_{33}^{inv} & A_{34}^{inv} \\ A_{41}^{inv} & A_{42}^{inv} & A_{43}^{inv} & A_{44}^{inv} \end{bmatrix} \tag{11}$$

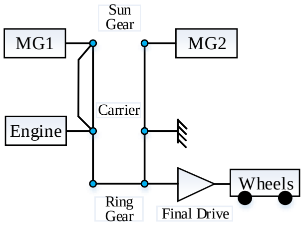


Figure 5. An example parallel mode in the Prius 2010 configuration

$$A^{-1}=\begin{bmatrix} A_{11}^{inv} & A_{12}^{inv} & A_{13}^{inv} & A_{14}^{inv} & A_{15}^{inv} \\ A_{21}^{inv} & A_{22}^{inv} & A_{23}^{inv} & A_{24}^{inv} & A_{25}^{inv} \\ A_{31}^{inv} & A_{32}^{inv} & A_{33}^{inv} & A_{34}^{inv} & A_{35}^{inv} \\ A_{41}^{inv} & A_{42}^{inv} & A_{43}^{inv} & A_{44}^{inv} & A_{45}^{inv} \\ A_{51}^{inv} & A_{52}^{inv} & A_{53}^{inv} & A_{54}^{inv} & A_{55}^{inv} \end{bmatrix}$$

$$A^*=\begin{bmatrix} A_{11}^{inv} & A_{12}^{inv} & A_{12}^{inv} & A_{13}^{inv} \\ A_{21}^{inv} & A_{22}^{inv} & A_{22}^{inv} & A_{23}^{inv} \\ A_{21}^{inv} & A_{22}^{inv} & A_{22}^{inv} & A_{23}^{inv} \\ A_{31}^{inv} & A_{32}^{inv} & A_{32}^{inv} & A_{33}^{inv} \end{bmatrix} \tag{12}$$

<u>*Step 2*</u>*: Refine A^* matrix*

For each row of A^*, if three entries are zero, the corresponding component has no connection with the other three components, i.e., the rest of the powertrain, then all the elements in the row are set to zero.

If both the 1st and the 2nd element of the 3rd and 4th row of A^* are 0, it means the MGs are neither connected with the engine nor the vehicle, they will not affect the function of the mode, so the 3rd and 4th rows are set to 0.

<u>*Step 3*</u>*: Define entries in the A^* matrix*

The four rows of the A^* matrix will be named as V_{veh}, V_{eng}, V_{MG1} and V_{MG2} respectively, and the elements of the V_{veh} row vector are named C_{veh}, C_{eng}, C_{MG1}, C_{MG2} for later use.

$$A^*=\begin{bmatrix} V_{veh} \\ V_{eng} \\ V_{MG1} \\ V_{MG2} \end{bmatrix}, V_{veh}=\begin{bmatrix} C_{veh} & C_{eng} & C_{MG1} & C_{MG2} \end{bmatrix} \tag{13}$$

If the first row of A^* is zero, the vehicle output is not affected by any powertrain component, making it infeasible (useless). In addition, vehicle modes with identical A^* matrices are deemed identical and only one mode will be kept.

2.4 Mode Classification

All feasible modes are classified into 14 classes shown in Table 1. They represent all possible mode types when one engine, one output shaft and two MGs are used in a double planetary gear powertrain system.

Step 1: Check the rank of the A^* matrix

Since each row of the A^* matrix represents the relationship between the torque input and a component's acceleration, rank reduction means that the acceleration of some component can be represented as a linear combination of the accelerations of other components. The DoF of the mode is determined by checking rank(A^*), which cannot be more than 3.

*Step 2: Formulate auxiliary matrixes_*

Six matrixes are generated for further rank analysis: $M_{VE} = [V_{veh}; V_{eng}]$, $M_{VMG1} = [V_{veh}; V_{MG1}]$, $M_{VMG2} = [V_{veh}; V_{MG2}]$, $M_{EMG1} = [V_{eng}; V_{MG1}]$, $M_{EMG2} = [V_{eng}; V_{MG2}]$, $M_{MG1MG2} = [V_{MG1}; V_{MG2}]$ and the ranks of those matrixes are denoted as r_{VE}, r_{VMG1}, r_{VMG2}, r_{EMG1}, r_{EMG2}, r_{MG1MG2}.

TABLE 1 MODE CLASSIFICATIONS AND THEIR CRITERIRA

	Mode Classification	**Criteria**
1	Series Mode	DoF=2, $C_{eng} = 0$, $V_{eng}(2) \neq 0$ $C_{MG1}C_{MG2} = 0$,
2	Compound Split (3 DoF)	DoF = 3
3	Compound Split (2 DoF)	DoF = 2, $C_{eng} \neq 0$, $C_{MG1}C_{MG2} \neq 0$, $r_{VMG1} = 2$, $r_{VE} = 2$, $r_{VMG2} = 2$, $r_{EMG1} = 2$, $r_{EMG2} = 2$
4	Input Split	DoF = 2, $C_{eng} \neq 0$, $r_{VMG1}\, r_{VMG2} = 2$,$C_{MG1}C_{MG2} \neq 0$
5	Output Split	DoF = 2, $C_{eng} \neq 0$, $r_{EMG1}\, r_{EMG2} = 2$,$C_{MG1}C_{MG2} \neq 0$
6	Parallel with EVT (Engine + 1MG)	DoF = 2, $C_{eng} \neq 0$, $C_{MG1}\, C_{MG2} = 0$, $C_{MG1}^2 + C_{MG2}^2 \neq 0$
7	Parallel with EVT (Engine + 2 MGs in serial)	DoF = 2, $C_{eng} \neq 0$, $C_{MG1}C_{MG2} \neq 0$, $r_{MG1MG2} = 1$
8	Engine Only (Fixed Gear)	DoF = 1, $C_{eng} \neq 0$ $C_{MG1}^2 + C_{MG2}^2 = 0$
9	Parallel with Fixed Gear (Engine + 2MGs, 2 DoF)	DoF = 2, $C_{eng} \neq 0$ $r_{VE} = 1$, $C_{MG1}C_{MG2} \neq 0$
10	Parallel with Fixed Gear (Engine + 2MGs, 1DoF)	DoF = 1, $C_{eng} \neq 0$ $C_{MG1}C_{MG2} \neq 0$
11	Parallel with Fixed Gear (Engine + 1MG, 1DoF)	DoF = 1, $C_{eng} \neq 0$ $C_{MG1}\, C_{MG2} = 0$, $C_{MG1}^2 + C_{MG2}^2 \neq 0$
12	EV (2MGs,2 DoF)	DoF = 2, $C_{eng} = 0$, $V_{eng}(2) = 0$
13	EV (2MGs,1 DoF)	DoF = 1, $C_{eng} = 0$ $C_{MG1}C_{MG2} \neq 0$
14	EV (1MG, 1 DoF)	DoF = 1, $C_{eng} = 0$ $C_{MG1}\, C_{MG2} = 0$, $C_{MG1}^2 + C_{MG2}^2 \neq 0$

The mode type for any given A^* matrix is then determined based on the criteria shown in Table 1.

For example, for the Prius 2010 shown in Figure 3, the rank of A^* is 2 with $r_{VMG1} \times r_{VMG2} = 2$ and $C_{eng} \neq 0$, $C_{MG1} \times C_{MG2} \neq 0$, which indicates it has an input-split mode.

CASE STUDY

In this section, we will choose the THS-II configuration used in Prius 2010 as a case study, and the vehicle parameters are shown in Table 2. Due to the page limitation, the detailed analysis on all possible clutch locations is omitted and will be presented in a follow-up paper.

TABLE 2 PARAMETERS OF THE VEHICLE USED IN THE CASE STUDY (BASED ON PRIUS 2010)

Component	**Parameters**
Engine	98 hp@5200rpm 105 lbft@4000rpm
P_{MG1max} (kW)	42
P_{MG2max} (kW)	60
FR	3.2
R_1:S_1	2.6
R_2:S_2	2.63
Vehicle mass (kg)	1450

In theory, 16 clutches may enable up to 2^{16}=65,536 modes. After applying the proposed screening process, we found there are 101 feasible and non-redundant modes for the THS-II configuration, as shown in Figure 6.

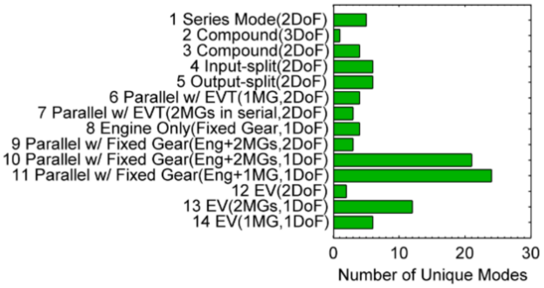


Figure 6. All feasible and non-redundant modes for the THS-II Configuration (used in Prius 2010)

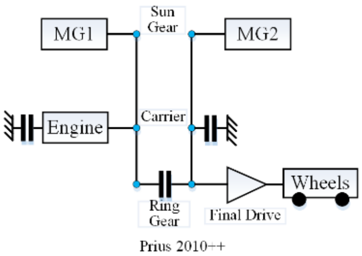
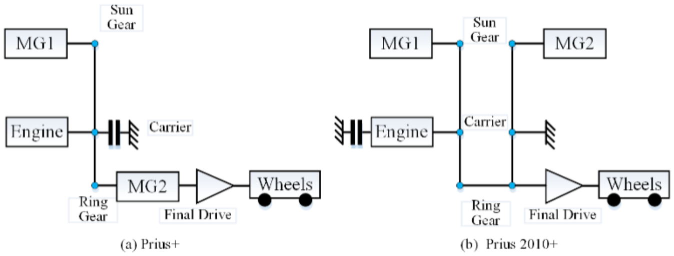


Figure 7 Proposed Prius 2010^{++} concept

Applying the near-optimal energy management strategy referred as PEARS [15], we identified a large number of designs with better fuel economy than the original Prius 2010. Among them, one of the best designs is presented in Figure 7, named as Prius 2010^{++}. It has five modes: 3 EV modes and 2 Hybrid Modes as shown in Table 3. It should be noted that in this specific design, Mode 1 and Mode 3 are special cases of Mode 2, while Mode 5 is a special case of Mode 4. Therefore, a simplified design with only 1 clutch and 2 modes is presented in Figure 8 (b), referred as Prius2010^{+} In this paper, which is very similar to a design named Prius^{+} , extension of the Prius 2004, proposed in [9], as shown in Figure 8 (a).



(a) Prius+ (b) Prius 2010+

Figure 8 Prius^{+} and Prius 2010^{+}

TABLE 3 MODES FOR PRIUS 2010^{++}

Mode #	**Class of Mode in Table 1**	**Description**
1	14	EV with MG1 only
2	13	EV with MG1 and MG2, 1 DoF
3	14	EV with MG2 only
4	4	Input-split
5	6	Parallel with EVT, Engine + MG1

To find a global optimal control execution, the Dynamic Programming technique is applied. The problem has 2 state and 3 control variables, as shown in Table 4.

TABLE 4 STATE AND CONTROL VARIABLES FOR PRIUS 2010^{+}

States & Control	**Variables**	**Range**
State 1	ω_{eng}	[0:100:5200] rpm
State 2	SOC	[0.4:0.01:0.7]
Control 1	T_{eng}	[0:5:140] Nm
Control 2	T_{MG1}	[-140:5:140] Nm
Control 3	Mode	{2,4}

In previous research [8][13][14], we assumed that the battery SOC drops from 0.55 to 0.43. This is about 0.9kWh of energy which is selected to exam both EV and hybrid performance. The optimized fuel consumption for Prius 2010 and Prius 2010^{+} for the FUDS cycle is shown in Table 5. It can be seen that the fuel economy difference between Prius 2010 and Prius 2010^{+} is about 12.6%.

TABLE 5 FUEL CONSUMPTION FOR PRIUS2010 AND PRIUS2010^{+} IN THE FUDS CYCLE

Design	**Prius 2010**	**Prius 2010^{+}**
Fuel Consumption(g)	108.3	94.7
Improvement	N/A	12.6%

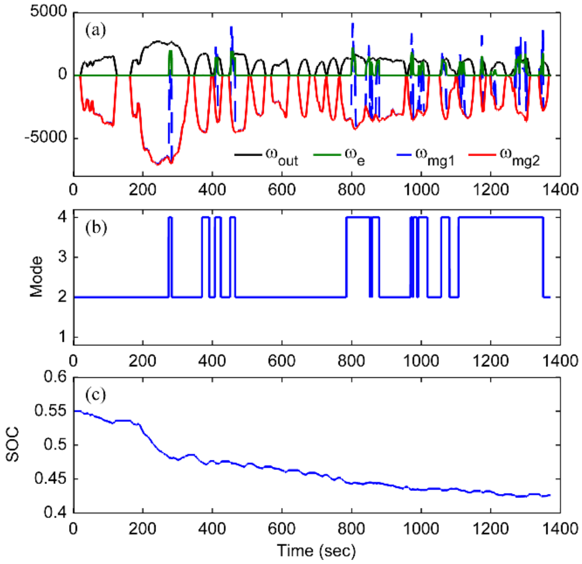


Figure 9 Component speed and Mode selection for Prius2010^{+}

The state and mode trajectories for Prius2010^{+} are shown in Figure 9. Conceptually, since the Power-split mode is the only hybrid mode for both Prius2010 and Prius2010^{+} designs, they should have very similar fuel economy when the engine is turned on (i.e., NOT operating in the EV modes). As a result, the benefit for Prius 2010^{+} must come from the EV mode. For Prius 2010, in its EV mode, the two motors must not drag the engine and therefore the generator speed must be controlled carefully to keep the engine speed close to zero, while the traction torque is mostly provided by MG2. This “speeder-torquer” role separation limits the operation of the electric machines, which is not the case for the Prius2010^{+} configuration since the engine is grounded and the torque distribution between MG1 and MG2 can be much more flexible.

To explain this more thoroughly and find more insights behind the number, a series of cases with different available battery energy are examined. Since it is rare that two designs consume exactly the same amount of battery energy, SOC correction is mandatory to compare the fuel economy between the two designs. We can select different SOC drop to reflect different battery energy consumption from charge depletion to charge sustaining scenario, as shown in Figure 10 and Table 6.

In Figure 10 (a), both Prius2010 and Prius2010^{+} vehicles are forced to run in their EV modes (i.e., the engine cannot be turned on), and we can find the improvement from pure EV driving is 5.8%. For Prius 2010, MG2 is the only electric machine that can be used in EV mode. For Prius 2010$^+$, on the other hand, MG1 has smaller size and lower torque range and can be fully utilized. Therefore, for the same operating condition, especially in less demanding cycles like FUDS, it is more likely for MG1 to operate in a more efficient area compared with MG2. In fact, for Prius 2010$^+$, MG1 provides most of the power in EV mode, leading to higher efficiency.

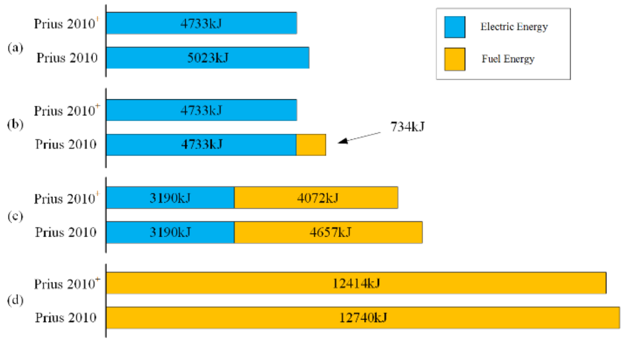


Figure 10 Energy analysis for different available battery energy

In Figure 10 (b), the available battery energy is just enough for Prius 2010$^+$ to finish the cycle without any engine operation. In this case, the fuel consumption improvement for Prius 2010$^+$ is infinite compared to Prius 2010. An interesting counterintuitive scenario is observed: In case (b), the total system energy consumption improvement for Prius 2010$^+$ is even higher than case (a), although all energy saved comes from EV mode. The reason is that the engine efficiency is much lower than that of the electrical system, therefore the total system energy consumption for Prius 2010 is significantly higher than Prius 2010$^+$ in this marginal condition.

In Figure 10 (c), it is for a typical driving condition with both battery and fuel energy consumed. As mentioned previously in this section, we assume that the battery SOC drops from 0.55 to 0.43, which lead to 3190kJ of battery energy consumption.

In Figure 10 (d), we fixed the initial SOC to the same as the final SOC, making both vehicles running with a charge sustaining strategy. It can be seen that the improvement we can get by adding clutches to the Prius 2010 design is only 2.6%, since Prius 2010 is already a well-designed HEV and the advantage of the additional EV mode cannot be fully realized in this charge sustaining case.

In summary, with the introduction of multiple-mode operation, not only the overall efficiency of the system will be increased due to the introduction of a more efficient EV mode (EV2 mode), but the difference in fuel consumption is amplified due to the small amount of fuels consumed.

Besides improving fuel economy, adding a clutch is also beneficial for drivability. Assuming there is no battery power limitation, it can be seen from Figure 11 that the output power for Prius 2010$^+$ is significantly higher, leading to 1.3 seconds faster 0 to 60 mph launching.

TABLE 6 FUEL CONSUMPTION COMPARISON BETWEEN PRIUS 2010 AND PRIUS 2010$^+$ WITH DIFFERENT BATTERY ENERGY CONSUMPTION IN THE FUDS CYCLE

Case	**Total Energy Consumption (kJ)**		**Energy saving**	**Fuel Consumption (g)**		**Fuel saving**
	Prius 2010	**Prius 2010$^+$**		**Prius 2010**	**Prius 2010$^+$**	
(a)	5023	4733	5.8%	0	0	N/A
(b)	5467	4733	13.4%	17.1	0	∞
(c)	7847	7262	7.7%	108.3	94.7	12.6%
(d)	12740	12414	2.6%	296.3	288.7	2.6%

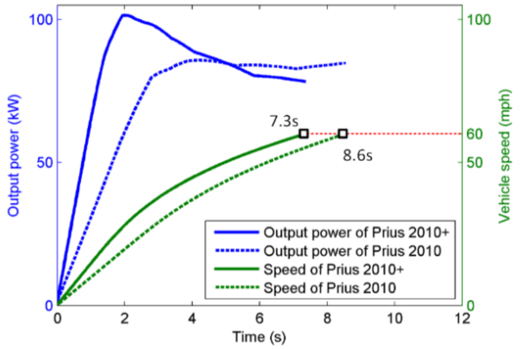


Figure 11 Output power and speed of Prius 2010 and Prius 2010$^+$ during 0 to 60 mph acceleration

CONCLUSION

A systematic and automated modeling procedure for power split powertrain using double PG is presented. We have developed mode screening and identification rules, so that infeasible and redundant modes are eliminated, which reduces the design pool significantly, making the design process more efficient. The configuration of Prius MY2010 is used as a case study. With the proposed method, we find that a total of 109 unique modes can be achieved when additional clutches are added. As a follow-up case study, we pick a special example which is evolved from [8]. With the help of Dynamic Programming technique, we find that by allowing three clutches to be added, a more complex powertrain configuration with 4 or even 2 modes make it possible to achieve better fuel economy and launching performance simultaneously. The fuel economy improvement for HEV design is limited. For PHEV, however, the benefits of multiple mode operation are much more significant.

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REFERENCES

[1] Part II Environment Protection Agency, Federal Register, Vol.77, No. 199, Book 2 of 2 Books, Pages 62623-63200, October 15, 2012.

[2] Electric drive vehicle sales, Electric Drive Transpiration Association, [online] http://electricdrive.org/index.php?ht=d/sp/i/20952/pid/20952 (Accessed: 20/09/2013)

[3] Alternative Fuels and Advanced Vehicles Data Center, Data, Analysis, and Trends: *Vehicle—HEV Sales by Model*, [online] 2010 http://www.afdc.energy.gov/afdc/data/vehicles.html (Accessed: 20/09/2013).

[4] M.R. Schmidt, "Two-Mode, compound-split electro-mechanical vehicular", U.S. patent 5 931 757, Aug, 1999

[5] K. Seo, "Powertrain for Hybrid Vehicle", U.S. patent 8 147 367, April, 2012

[6] K. Rahman, M. Anwar, "The Voltec 4ET50 Electric Drive System", *SAE International*, vol.4, no.1 323-337. June, 2011

[7] B. Si, "Reconfiguration hybrid powertrain", U.S. patent 0 319 211, Dec, 2011

[8] X. Zhang, C. Li, D. Kum, H. Peng, "Prius+ and Volt-: Configuration Analysis of Power-Split Hybrid Vehicles with a Single Planetary Gear", *IEEE Transactions on Vehicular Technology*, vol. 61, Issue 8, pp. 3544-3552, 2012.

[9] J. Liu and H. Peng, "A systematic design approach for two planetary gear split hybrid vehicles," *Vehicle System Dynamics*, vol. 48, pp. 1395-1412, 2010.

[10] C.-T. Li and H. Peng, "Optimal configuration design for hydraulic split hybrid vehicles," in *American Control Conference*, Baltimore, MD, 2010, pp. 5812-5817.

[11] M. Schmidt, "Two-mode, split power, electro-mechanical transmission," U.S. Patent 5 577 973, 1996.

[12] H. Benford and M. Leising, "The lever analogy: a new tool in transmission analysis," *SAE Paper*, 810102, 1981.

[13] X. Zhang, H. Peng, J. Sun, "A Near-optimal Energy Management Strategy For Rapid Component Sizing of Power Split Hybrid Vehicles with Multiple Operating Modes", in *American Control Conference*, Washington DC, pp. 5972-5977, June 2013.

[14] X. Zhang, C.Li, D. Kum, H. Peng, J. Sun, "Configuration Analysis for Power Split Hybrid Vehicles with Multiple Operating Modes", in *AVEC Conference*, Seoul, Korea, 2012.

[15] X. Zhang, H. Peng, J. Sun, "A Near-optimal Energy Management Strategy For Rapid Component Sizing of Power Split Hybrid Vehicles with Multiple Operating Modes", in *American Control Conference*, Washington DC, pp. 5972-5977, June 2013.