Robust Longitudinal Control of Multi-Vehicle Systems—A Distributed H-Infinity Method

Shengbo Eben Li, Feng Gao, Keqiang Li, Le-Yi Wang, *Fellow, IEEE*, Keyou You, and Dongpu Cao

***Abstract*—The platooning of automated vehicles has the potential to significantly benefit road traffic. This paper presents a distributed H_∞ control method for multi-vehicle systems with identical dynamic controllers and rigid formation geometry. After compensating for the powertrain nonlinearity, the node dynamics in a platoon is mathematically described by a multiplicative uncertainty model. The platoon control system is then decomposed into an uncertain part and a diagonal nominal system through linear transformation and eigenvalue decomposition of the information-exchange-topology matrix. Robust stability, string stability, and distance tracking performance of the designed platoons are analyzed theoretically under the decoupled H_∞ framework. A comparative simulation with non-robust controllers is used to demonstrate the effectiveness of this method.**

***Index Terms*—Automated vehicle, platoon control, distributed control, robustness, string stability.**

I. INTRODUCTION

DURING the past decades, the increasing traffic demand has brought a heavy burden on the existing transportation infrastructure and sometimes leads to heavily congested road networks [1]. The platooning of automated vehicles has the potential to significantly improve traffic capacity, safety and fuel efficiency, which is attracting increasing attention of both academia and automotive industry [2]. The goal of platoon control is to ensure that vehicles maintain desired speeds according to pre-specified inter-vehicle spacing policies. To the best of our knowledge, the earliest practices on platoon control can date back to the PATH project in California [3]. Since then, many topics have been discussed, including spacing policies [4], [5], information exchange topologies [6], [7], powertrain dynamics and communication delay [8], [9], homogeneity and heterogeneity [10], *etc*. Advanced control methods have been introduced into platoon automation to achieve systematic performance. For instance, Liang and Peng [11] proposed an optimal control strategy for the upper-level controller to guarantee string stability. Li *et al.* [12] has presented a periodic control method for homogeneous platoon for the purpose of minimizing overall fuel consumption. More recently, some platoons have been demonstrated in the real world, including the Grand Cooperative Driving Challenge (GCDC) in the Netherlands, the SARTRE project in Europe, and the Energy-ITS project in Japan, *etc* [2].

Summarizing the existing studies, there are mainly two ways to design platoon control systems. One is to consider all vehicles in a platoon as a whole, and then design the decentralized controllers. For example, Guo and Yue [13] designed full-state feedback controllers for a group of vehicles considering their parametric uncertainties, communication delay, and disturbance attenuation. The main shortcoming of this design is that such controllers only work for the platoon with the fixed length and communication topology. The other way is to decompose a platoon into a series of connected local subsystems and then apply distributed controllers to each subsystem. For example, Stankovic *et al.* [14] used the inclusion principle to decompose a platoon into locally decoupled subsystems, for which decentralized overlapping controllers were designed. Dunbar and Caveney [15] proposed a distributed receding horizon controller for vehicular platoons and derived the sufficient condition of asymptotic stability. Herman *et al.* [16] proposed an asymmetric bidirectional controller to ensure string stability without the information of lead vehicle. These studies are only applicable for homogeneous platoons, i.e., all nodes have the identical dynamics [17]. In practice, a platoon in real world may consist of different types of vehicles, or the same types of vehicles but with different parameters. Moreover, the accurate models of engine, clutch, gearbox, tire and braking system are not available; and the dynamics of each node also varies with its environment and road.

The robustness of platoon control systems is an important, but less investigated topic. One existing way to handle the model mismatch in the vehicle dynamics is to employ

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so-called lower-level control to generate approximately consistent and accurate input-output behavior for node dynamics [18]. This often results in a more complex platoon control system. In addition, such issues as heterogeneity in node dynamics and string stability of platoon with a complex information topology are still not accommodated in this design. Therefore, a distributed $\boldsymbol{H}_\infty$ method is quite necessary to comprehensively consider the robust stability, string stability and tracking performance of a platoon. Lin *et al.* [20] have investigated the distributed robust control considering external disturbance and parameter uncertainty in the interaction topologies for the directed networks of one-order agents. Tian and Liu [21] studied the robustness of symmetric systems with asymmetric interconnection perturbation, and the bound on the largest singular value of perturbation matrix was mathematically derived as the consensus condition. Khoo *et al.* [22] studied the finite time tracking control considering input disturbance for second-order multi-robot systems. Zheng *et al.* [23] has presented a distributed model predictive control algorithm for heterogeneous vehicle platoons with unidirectional topologies and a priori unknown desired set point. An equality based terminal constraint is proposed to ensure the closed-loop stability, which enforces the terminal states of each node in the predictive horizon equal to the average of its neighboring states. Han *et al.* [24] investigated the robust consensus problem with uncertain interaction topologies whose communication links are weighted by polynomial functions of an uncertain vector constrained in a semi-algebraic set. Huang *et al.* [25] considered the consensus problem subjected to the node model uncertainty, polytopic topology uncertainty and external disturbances for homogeneous multi-agent systems. Li *et al.* [26] presented a low dimensional solution to distributed robust control problems considering $\boldsymbol{H}_\infty$ bounded node model uncertainty with undirected communication topologies.

The main purpose of this paper is to provide a distributed robust control method for heterogeneous vehicular platoons to balance the performance of robust stability, disturbance attenuation and string stability. By applying the eigenvalue decomposition to topological matrix, a platoon control system with identical distributed controllers is decomposed into two parts: a bounded uncertain part and a diagonal nominal part. The robust stability, tracking performance, and string stability are analyzed theoretically in the $\boldsymbol{H}_\infty$ control framework.

The rest of this paper is organized as follows: Section II describes the studied heterogeneous platoon. Section III introduces the decoupling and synthesis process. Section IV validates this method by simulation. Section V concludes the paper.

II. Problem Description

This paper considers a heterogeneous vehicle platoon in Fig. 1. The platoon includes $N+1$ vehicles (or nodes), *i.e.*, a leader (indexed as 0) and N followers (indexed by i accordingly). The dynamics of each vehicle is inherently nonlinear, which include engine, transmission, final gear, brake system, aerodynamics drag, tire friction, and rolling resistance, *etc*.

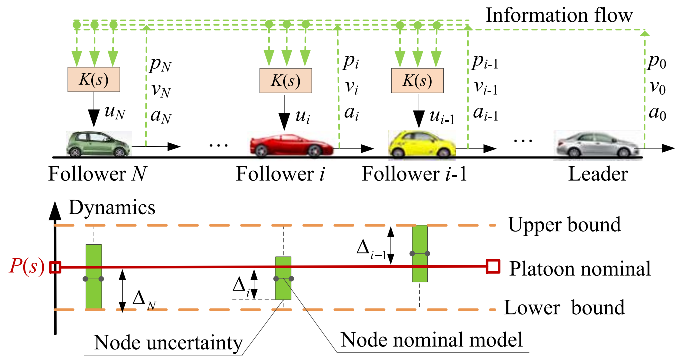

Fig. 1. Heterogeneous vehicular platoon and uncertainty.

The platoon can have different information flow topologies, either frontal radar-based or wireless communication-based, including predecessor following type, predecessor-leader following type, *etc*. [27], [29].

Let $\mathbb{R}^{N\times N}$ be the set of $N\times N$ real matrices and $\mathbf{RH}_\infty$ be the matrix set composed of real rational transfer functions, whose poles lie in the open left-half complex plane. The notation s is the Laplace operator and $j=\sqrt{-1}$ is the imaginary unit. The notation $\mathbf{1}_N \in \mathbb{R}^N$ denotes the vector with one being its elements. $diag(\boldsymbol{A}_1,\ldots,\boldsymbol{A}_N)$ denotes a block diagonal matrix with matrices $\boldsymbol{A}_i$, $i=1,\ldots,N$ being its diagonal blocks and $rank(\cdot)$ denotes the rank of a matrix. $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{O}$ denotes a zero matrix. Notions $||\cdot||_2$ and $||\cdot||_\infty$ are the $\mathbf{L}_2$ norm of signals and its induced norm, *i.e.*, the $\boldsymbol{H}_\infty$ norm of a system, respectively. The notation $\sigma(.)$ denotes a singular value of a matrix, among which $\overline{\sigma}(.)$ and $\underline{\sigma}(.)$ are the maximum and minimum ones in magnitude. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

A. Models of Dynamics and Topology

The platoon control includes two tasks: (1) to implement control of platoon formation, stabilization and dissolution; and (2) to perform control for throttle/brake actuators. These naturally lead to a hierarchical control structure, including an upper controller and a lower controller. The upper one is for platoon motion management to retain safe and string-stable operations, while the lower one is for accurate acceleration tracking. This paper focuses on the upper level and the nonlinearities of powertrain dynamics are handled by a lower level controller [18]. Therefore, combined with the lower level controller, the dynamics of node i can be simplified to be a multiplicative uncertainty model [19]:

$$
\begin{aligned}
&p_i(s)=\frac{1}{s}v_i(s),\quad v_i(s)=\frac{1}{s}a_i(s)\\
&a_i(s)=P(s)\left[1+\Omega(s)\Delta_i\right]u_i(s),\quad i=1,\cdots,N \qquad (1)
\end{aligned}
$$

where $p_i(s), v_i(s), a_i(s), u_i(s)$ is the position, velocity, acceleration and desired acceleration, respectively; $P(s)$ is the nominal model of all vehicles. The model uncertainty is described by a normalized uncertain term $\Delta_i \in \mathbf{RH}_\infty$, satisfying $\|\Delta_i\|_\infty \le 1$, with the weighting function $\Omega(s)\in\mathbf{RH}_\infty$. Note that the uncertainty arises from two sources (shown by Fig. 1): (1) The modeling error of each vehicle, including

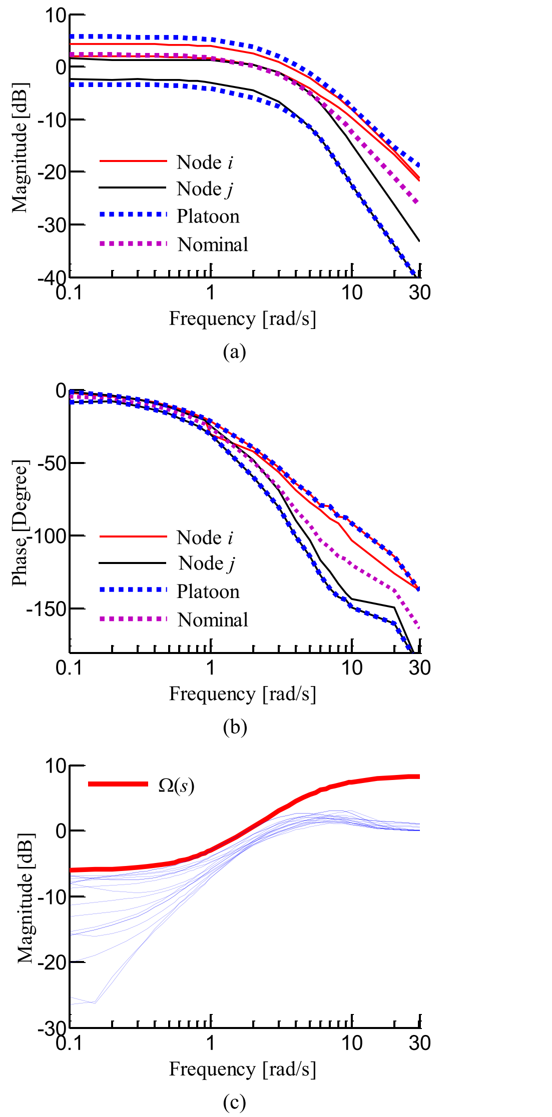

Fig. 2. Uncertainties of nodes in the platoon. (a) Magnitude range. (b) Phase range. (c) Weighting function of platoon uncertainty.

linearization error, parameter variation, high-order dynamics, *etc.*; (2) The dynamical difference among nodes caused by so called platoon heterogeneity, which means that the platoon heterogeneity is regarded as a part of model uncertainty.

A passenger car is used to study the node dynamics. Its powertrain contains a 1.6-L gasoline engine, a torque converter, a four-speed automatic transmission, two driving and two driven wheels, and a hydraulic braking system [19]. The nonlinearities of powertrain are compensated by an inverse model (39). Combined with the lower level controller, the transfer function of node dynamics is estimated by Matlab ***ident*** toolbox. Fig. 2 (a) and (b) show an example of the dynamics range of practical nodes, and Fig. 2 (c) is the corresponding model uncertainties and the weighting function.

The algebraic graph is adopted to describe various information flow topologies [6], [27]. The graph is encoded into a Laplacian matrix and a pinning matrix. The Laplacian matrix $\boldsymbol{L}$ actually describes the directional communication among followers, and is defined as

$$\boldsymbol{L} = [l_{ik}] \in \mathbb{R}^{N\times N}, \quad l_{ik} = \begin{cases} -m_{ik}, & \text{if } k \neq i \\ \sum_{r=1,r\neq i}^{N} m_{ir}, & \text{if } k = i \end{cases} \tag{2}$$

where $m_{ik} = 1$ if the follower i receives the information from k; otherwise, $m_{ik} = 0$. The pinning matrix $\boldsymbol{P}$ represents the directional connection from the leader to each follower, defined as

$$\boldsymbol{P} = diag\,(g_1, \cdots, g_N) \in \mathbb{R}^{N\times N} \tag{3}$$

where$g_i = 1$ if the follower i receives information from the leader; otherwise, $g_i = 0$. The node $k \in \{0, \cdots, N\}$, including both the leader and followers, is said to be a neighbor of node i if it directly sends information to node i. All the neighbors of i constitutes the neighbor set of i, defined as $\mathbb{N}_i$:

$$\mathbb{N}_i = \begin{cases} \{k\,|m_{ik} = 1\}, & \text{if } g_i = 0 \\ \{k\,|m_{ik} = 1\} \cup \{0\}, & \text{if } g_i \neq 0 \end{cases} \tag{4}$$

According to [27, Lemma 2], $\boldsymbol{L} + \boldsymbol{P}$ is nonsingular and all followers are either directly pinned to the leader or have a path connecting the vehicle to another vehicle pinned to the leader.

B. The Closed-Loop LFT Formation

The objective of platoon control in the upper level is to track the leader while maintaining a rigid formation governed by a given spacing policy between any two consecutive vehicles, *i.e.*

$$\begin{cases} v_i\,(t) = v_0\,(t) \\ p_i\,(t) - p_k\,(t) = d_{ik} \end{cases}, \; i = 1, \cdots, N \tag{5}$$

where d_{ik} is the desired constant distance between the vehicle $i$and k. The constant distance (CD) policy is used, which is independent of vehicle velocity, thus capable of leading to a very high traffic capacity. Other policies, *e.g.* constant time-headway, can also be used to better accord with driver behaviors [4].

Considering that (1) The nominal parts of each node model are the same and intuitively each node may be controlled finely by the same form of controller; (2) Each vehicle only receives information from neighboring vehicles, which results in the difficulty to modify the controller along platoon, the distributed controllers are often designed to have identical feedback gain, as studied in [16]. The following state feedback control law is used to control the platoon:

$$\begin{aligned} u_i\,(t) = \sum_{k\in \mathbb{N}_i} \{&K_p\,[p_i\,(t) - p_k\,(t) - d_{ik}] \\ &+ K_v\,[v_i\,(t) - v_k\,(t)] + K_a\,[a_i\,(t) - a_k\,(t)]\} \end{aligned} \tag{6}$$

where K_p, K_v and K_a are the feedback gains. Thus, a communication link, if it exists, is assumed to be perfect in the sense that we ignore the effects such as quantization, packet loss and delay. To synthesize the controller under the $\boldsymbol{H}_\infty$ control framework, (1) is rewritten into the LFT (Linear Fractional

Transformation) form to decompose its nominal and uncertain parts:

$$\begin{aligned} p_i(s) &= \frac{1}{s^2}P(s)u_i(s) + \frac{1}{s^2}w_i(s) \\ z_i(s) &= \boldsymbol{\Omega}(s)P(s)u_i(s) \\ w_i(s) &= \Delta_i z_i(s), i = 1, \cdots, N \end{aligned} \tag{7}$$

where $w_i(s), z_i(s)$ are the disturbance and normalized excitation signal of Δ_i respectively. Let us introduce new error variable $e_i(t) = p_i(t) - p_0(t) - d_{i0}$, (6) is rewritten as:

$$u_i(s) = K(s)\sum_{k\in\mathbb{N}_i}[e_i(s) - e_k(s)] \tag{8}$$

where $K(s) = K_p + K_v s + K_a s^2$. Combining (2), (3), (7) and (8) yields the LFT representation of platoon control systems:

$$\begin{aligned} &\boldsymbol{E}(s) = \frac{1}{s^2}P(s)\boldsymbol{U}(s) + \frac{1}{s^2}\boldsymbol{W}(s) - \mathbf{1}_N p_0(s) - \frac{1}{s}\boldsymbol{\Gamma}_0 \\ &\boldsymbol{Z}(s) = \boldsymbol{\Omega}(s)P(s)\boldsymbol{U}(s), \boldsymbol{U}(s) = K(s)(\boldsymbol{L}+\boldsymbol{P})\boldsymbol{E}(s) \\ &\boldsymbol{W}(s) = \boldsymbol{\Delta}\boldsymbol{Z}(s) \end{aligned} \tag{9}$$

where $\boldsymbol{E}(s) = \begin{bmatrix} e_1(s) \\ \vdots \\ e_N(s) \end{bmatrix}$, $\boldsymbol{U}(s) = \begin{bmatrix} u_1(s) \\ \vdots \\ u_N(s) \end{bmatrix}$, $\boldsymbol{\Gamma}_0 = \begin{bmatrix} d_{10} \\ \vdots \\ d_{N0} \end{bmatrix}$, $\boldsymbol{W}(s) = \begin{bmatrix} w_1(s) \\ \vdots \\ w_N(s) \end{bmatrix}$, $\boldsymbol{Z}(s) = \begin{bmatrix} z_1(s) \\ \vdots \\ z_N(s) \end{bmatrix}$, and $\boldsymbol{\Delta} = diag(\Delta 0_1, \cdots, \Delta_N)$.

Since $\|\Delta_i\|_\infty \leq 1, i = 1, \cdots, N$, the property $\|\boldsymbol{\Delta}\|_\infty \leq 1$ holds according to [28]. The LFT format (9) actually separates the nominal and uncertain parts, which allows us to design the distributed controllers under the $\boldsymbol{H}_\infty$ control framework. Due to the diagonal structure, the elements Δ_i in $\boldsymbol{\Delta}$ have no interaction, which means that the node uncertainties are decoupled in the platoon control system.

III. Distributed $\boldsymbol{H}_\infty$ Control of Platoon

If the platoon length and communication topology are known beforehand and will not change during driving, many methods can be used to design the $\boldsymbol{H}_\infty$ controller based on (8) and (9). In this section, a decoupling method is proposed to convert the $\boldsymbol{H}_\infty$ controller synthesis problem of coupled platoon system to that of several decoupled sub-systems. Firstly some lemmas are introduced.

A. Lemmas

Lemma 1: If there exists a diagonal matrix $\boldsymbol{D}>0$, and two scalars $\eta_1 > 0$ and $\eta_2 > 0$, such that

$$\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D} - \eta_1^2\left(\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right)^{-1} < 0 \; and \; \eta_2^2\left(\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right)^{-1} - \boldsymbol{D}^{\mathrm{T}}\boldsymbol{D} < 0 \tag{10}$$

then $\bar{\sigma}(\boldsymbol{D}\boldsymbol{X}) < \eta_1$ and $\sigma(\boldsymbol{D}\boldsymbol{X}) > \eta_2$.

Proof: Multiplying (10) with $\boldsymbol{X}$ and $\boldsymbol{X}^{\mathrm{T}}$ on both sides yields $\eta_2^2\boldsymbol{I} < (\boldsymbol{D}\boldsymbol{X})^{\mathrm{T}}\boldsymbol{D}\boldsymbol{X} < \eta_1^2\boldsymbol{I}$. ■

$$\begin{aligned} \boldsymbol{y}_1(s) &= \boldsymbol{G}_1(s)\left[\boldsymbol{u}_1(s) \pm \boldsymbol{y}_2(s)\right] \\ \boldsymbol{y}_2(s) &= \boldsymbol{G}_2(s)\left[\boldsymbol{u}_2(s) \pm \boldsymbol{y}_1(s)\right] \end{aligned} \tag{11}$$

where $\boldsymbol{u}_1(s), \boldsymbol{u}_2(s), \boldsymbol{y}_1(s), \boldsymbol{y}_2(s), \boldsymbol{G}_1(s), \boldsymbol{G}_2(s) \in \boldsymbol{RH}_\infty$. It is stable if $\gamma_1\gamma_2 < 1$, where $\gamma_1 = \|\boldsymbol{G}_1(s)\|_\infty$ and $\gamma_2 = \|\boldsymbol{G}_2(s)\|_\infty$.

Lemma 3: For the graph $\boldsymbol{L}+\boldsymbol{P}$, if $\boldsymbol{P} = \boldsymbol{I}$ then:

(a) 1 is one of its eigenvalues and $\mathbf{1}_N$ is the corresponding eigenvector;

(b) If 1 is an m repeated eigenvalue, there also exist m linear independent eigenvectors;

(c) Clustering all eigenvalues which equal 1 together, the decomposition in (15) can be redefined as

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \bar{\Lambda} \end{bmatrix}, \boldsymbol{X}^{-1} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix}, \boldsymbol{X} = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \tag{12}$$

where $\bar{\Lambda} \in \mathbb{R}^{(N-m)\times(N-m)}$ is composed of $\lambda_i \neq 1$. Then we have

$$\begin{aligned} &\bar{X}_1 X_1 = \boldsymbol{I}, \bar{X}_2 X_2 = \boldsymbol{I}, \bar{X}_2 X_1 = \boldsymbol{O}, \bar{X}_1 X_2 = \boldsymbol{O} \\ &X_1\bar{X}_1 + X_2\bar{X}_2 = \boldsymbol{I}, \bar{X}_2\mathbf{1}_N = \boldsymbol{O}. \end{aligned} \tag{13}$$

Proof: $\boldsymbol{P} = \boldsymbol{I}$ implies that all followers communicate with the leader. Then, we have $(\boldsymbol{L}+\boldsymbol{P})\mathbf{1}_N = \boldsymbol{L}\mathbf{1}_N + \mathbf{1}_N = \mathbf{1}_N$. And (3.a) is proved.

Next, we have $|\lambda\boldsymbol{I} - \boldsymbol{P} - \boldsymbol{L}| = |(\lambda - 1)\boldsymbol{I} - \boldsymbol{L}| = 0 \Rightarrow \lambda = \lambda_L + 1$, where λ_L is the eigenvalue of $\boldsymbol{L}$. Therefore, $\boldsymbol{L}+\boldsymbol{P}$ has the m-repeated eigenvalue 1, if and only if 0 is an m-repeated eigenvalue of $\boldsymbol{L}$ and $rank(\boldsymbol{L}) = N - m$. This leads to the fact that there exist m linearly independent vectors $\tilde{X} \in \mathbb{R}^N$ satisfying $\boldsymbol{L}\tilde{X} = \boldsymbol{O}$. Since $\boldsymbol{L}\tilde{X} = \boldsymbol{O} \Leftrightarrow (\boldsymbol{L}+\boldsymbol{P})\tilde{X} = \tilde{X}$, $\boldsymbol{L}+\boldsymbol{P}$ has m linearly independent eigenvectors and (3.b) is proved. Since $\boldsymbol{X}^{-1}\boldsymbol{X} = \boldsymbol{X}\boldsymbol{X}^{-1} = \boldsymbol{I}$, Eq. (13) is derived except for $\bar{X}_2\mathbf{1}_N = \boldsymbol{O}$. According to (3.a) and (12), we have

$$(\boldsymbol{L}+\boldsymbol{P})\mathbf{1}_N = X_1\bar{X}_1\mathbf{1}_N + X_2\bar{\Lambda}\bar{X}_2\mathbf{1}_N = \mathbf{1}_N. \tag{14}$$

Multiplying (14) with $\bar{X}_2$ on both sides yields $\left(\bar{\Lambda} - \boldsymbol{I}\right)\bar{X}_2\mathbf{1}_N = \boldsymbol{O}$. This equation holds if and only if $\bar{X}_2\mathbf{1}_N = \boldsymbol{O}$, because $\bar{\Lambda}$ is composed of $\lambda_i \neq 1$. Then, (3.c) is proved. ■

B. Structural Decoupling of Platoon System

The structural decoupling for platoon control problem is obtained by a linear transformation. This transformation actually converts the problem with coupled controllers to a problem with decoupled controllers. Meanwhile, the coupling feature is moved to the uncertainty, which was originally decoupled. As shown in Fig. 3 (a), the controllers of (9) are coupled by $\boldsymbol{L}+\boldsymbol{P}$, while its uncertainties are decoupled. After the linear transformation, a new platoon control problem arises with decoupled controllers, but coupled uncertainties, shown in Fig. 3 (b).

The topological matrix $\boldsymbol{L}+\boldsymbol{P}$ has the following generalized eigen-decomposition:

$$\boldsymbol{L}+\boldsymbol{P} = \boldsymbol{X}\boldsymbol{\Lambda}\boldsymbol{X}^{-1} \tag{15}$$

where $\boldsymbol{X} \in \mathbb{R}^{N\times N}$ is composed of corresponding eigenvectors and generalized eigenvectors of $\boldsymbol{L}+\boldsymbol{P}$, $\boldsymbol{\Lambda} = diag(\boldsymbol{\Lambda}_1, \cdots, \boldsymbol{\Lambda}_n) \in \mathbb{R}^{N\times N}$ is a block diagonal matrix and $\sum_{i=1}^{n} rank(\boldsymbol{\Lambda}_i) = N$. Each diagonal block $\boldsymbol{\Lambda}_i$ is determined by eigenvalue λ_i of $\boldsymbol{L}+\boldsymbol{P}$, which has three possibilities:

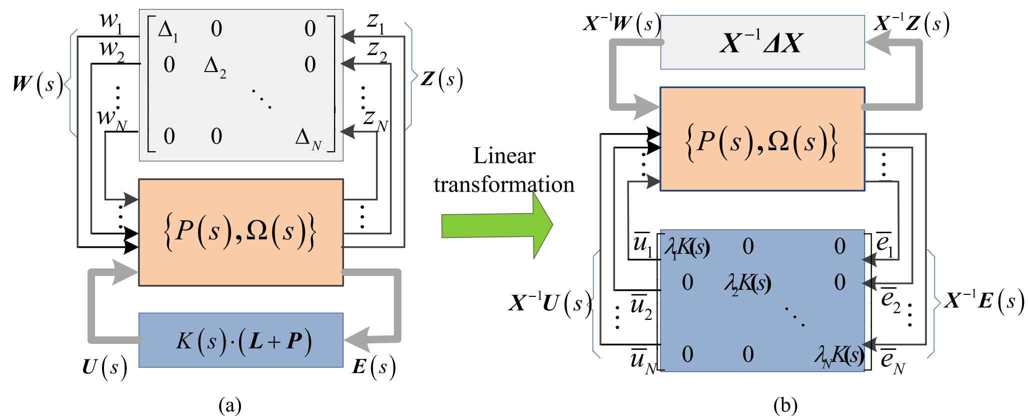

Fig. 3. Decoupling concept for robust platoon control. (a) Coupled controllers, but decoupled uncertainties. (b) Decoupled controllers, but coupled uncertainties.

(1) If $\lambda_i \in \mathbb{R}$ is a scalar and has a linearly independent eigenvector, then $\mathbf{\Lambda}_i = \lambda_i$;

(2) If $\lambda_i \in \mathbb{R}$ is an m-repeated scalar eigenvalue and only has one linearly independent eigenvector, then $\mathbf{\Lambda}_i \in \mathbb{R}^{m\times m}$ is in Jordan form: $\mathbf{\Lambda}_i = \begin{bmatrix} \lambda_i & 1 & \\ & \ddots & 1 \\ & & \lambda_i \end{bmatrix}$;

(3) If $\lambda_i = \alpha \pm \beta j$ is a simple eigenvalue, then $\mathbf{\Lambda}_i = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}$;

(4) If $\lambda_i = \alpha \pm \beta j$ is an m -repeated complex eigenvalue and only has a pair of conjugate complex eigenvectors, then

$$\mathbf{\Lambda}_i = \begin{bmatrix} \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} & \boldsymbol{I} & \\ & \ddots & \boldsymbol{I} \\ & & \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \end{bmatrix} \in \mathbb{R}^{2m\times 2m}.$$

Substituting (15) to (9), the decoupled platoon control system is obtained:

$$\begin{aligned} &\bar{E}(s) = \frac{1}{s^2}\left[P(s)\bar{U}(s) + \bar{W}(s)\right] - \left[\boldsymbol{X}^{-1}\mathbf{1}_N p_0(s) + \frac{1}{s}\bar{\Gamma}_0\right] \\ &\bar{Z}(s) = \mathbf{\Omega}(s)P(s)\bar{U}(s), \quad \bar{U}(s) = K(s)\mathbf{\Lambda}\bar{E}(s) \\ &\bar{W}(s) = \bar{\Delta}\bar{Z}(s) \end{aligned} \tag{16}$$

The variables in (16) are linear transformation of its original counterparts:

$$\begin{aligned} &\bar{Z}(s) = \boldsymbol{X}^{-1}\boldsymbol{Z}(s), \bar{U}(s) = \boldsymbol{X}^{-1}\boldsymbol{U}(s), \bar{E}(s) = \boldsymbol{X}^{-1}\boldsymbol{E}(s) \\ &\bar{W}(s) = \boldsymbol{X}^{-1}\boldsymbol{W}(s), \bar{\Gamma}_0 = \boldsymbol{X}^{-1}\mathbf{\Gamma}_0, \bar{\Delta} = \boldsymbol{X}^{-1}\mathbf{\Delta}\boldsymbol{X} \end{aligned} \tag{17}$$

Note that the certain part of system (16) is a block diagonal system and the interconnection among followers arising from the communication topology is decoupled. The platoon control problem is converted to a problem to control n subsystems whose individual dimension is greatly reduced.

The key point is on how to reduce the enlargement of converted uncertainty $\bar{\Delta}$ compared to original uncertainty $\mathbf{\Delta}$. When the topology is undirected or symmetric "look-ahead & look-back" type, $\boldsymbol{L}+\boldsymbol{P}$ is diagonally symmetric. It has N real eigenvalues and N linearly independent eigenvectors, and $\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{I}$, which leads to $\|\bar{\Delta}\|_\infty = \|\mathbf{\Delta}\|_\infty$ [28]. In this case, this technique does not enlarge the converted uncertainty, and no conservativeness is introduced; Otherwise, $\bar{\Delta}$ may be enlarged compared to $\mathbf{\Delta}$. In this case, it is possible to minimize the enlargement in uncertainty. Considering $\mathbf{\Delta}$ is diagonal, a proper diagonal matrix $\boldsymbol{D} = diag(\boldsymbol{D}_1, \cdots, \boldsymbol{D}_n) \in \mathbb{R}^{N\times N}$ can be selected by minimizing η_1/η_2 according to **Lemma 1**, in which η_1 and η_2 are two design parameters. In addition, the form of $\boldsymbol{D}_i$ is required to satisfy the following conditions: (a) $rank(\boldsymbol{D}_i) = rank(\mathbf{\Lambda}_i)$; (b) If $rank(\mathbf{\Lambda}_i) > 1$, $\boldsymbol{D}_i = \delta_i\boldsymbol{I}$. Then, the upper bound of $\|\bar{\Delta}\|_\infty$ can be estimated by

$$\|\bar{\Delta}\|_\infty = \left\|\boldsymbol{X}^{-1}\boldsymbol{D}^{-1}\mathbf{\Delta}\boldsymbol{D}\boldsymbol{X}\right\|_\infty \le \bar{\sigma}(\boldsymbol{D}\boldsymbol{X})/\sigma(\boldsymbol{D}\boldsymbol{X}) \le \beta \tag{18}$$

where $\beta = \eta_1/\eta_2 \ge 1$. The minimization can be numerically solved by LMI, which is helpful to reduce the enlargement in uncertainty.

C. Distributed H_∞ Controller Design

Substituting $s^2 p_0(s) = a_0(s)$ and $s\boldsymbol{X}^{-1}\mathbf{\Gamma}_0 = \boldsymbol{O}$ to (16), the platoon system becomes the following block diagonal system:

$$\begin{aligned} &\bar{E}(s) = \begin{bmatrix} \boldsymbol{\tau}_d(\mathbf{\Lambda}_1) & & \\ & \ddots & \\ & & \boldsymbol{\tau}_d(\mathbf{\Lambda}_n) \end{bmatrix}\left[\bar{W}(s) - \boldsymbol{X}^{-1}\mathbf{1}_N a_0(s)\right] \\ &\bar{Z}(s) = \mathbf{\Omega}(s)P(s)K(s)\mathbf{\Lambda}\bar{E}(s), \quad \|\bar{W}(s)\|_2 \le \beta\|\bar{Z}(s)\|_2 \end{aligned} \tag{19}$$

where $\boldsymbol{\tau}_d(\mathbf{\Lambda}_i) = \left[s^2\boldsymbol{I} - P(s)K(s)\mathbf{\Lambda}_i\right]^{-1}$. Then by using the **Small Gain Theorem** (Lemma 2), it can be concluded that if there exits $K(s) = K_s + K_v s + K_a s^2$ and constants $\gamma, \alpha > 0$, such that

$$\|\mathbf{\Omega}(s)P(s)K(s)\mathbf{\Lambda}_i\boldsymbol{\tau}_d(\mathbf{\Lambda}_i)\|_\infty \le \gamma < 1/\beta$$

and

$$\|W_p(s)\boldsymbol{\tau}_d(\mathbf{\Lambda}_i)\|_\infty \le \alpha, i = 1, \cdots n \tag{20}$$

where $W_p(s)$ is the weighting function of performance, and then the decoupled system (19) is robust stable and $\bar{E}(s)$

satisfies

$$\left\| W_p(s)\,\bar{E}(s) \right\|_2 \le \frac{\alpha}{1-\gamma\beta} \left\| \boldsymbol{X}^{-1}\mathbf{1}_N a_0(s) \right\|_2 . \tag{21}$$

Since the linear transformation does not change the stability of a system, the platoon system controlled by $K(s)$ is also robustly stable. Furthermore, there are some numerical methods to find such $K(s)$ satisfying (20). For example, by using the common Lyapunov method, the requirements of (20) can be converted to multiple LMIs with the output-feedback $\boldsymbol{H}_\infty$ control structure [28]. Though $\left\|\bar{E}(s)\right\|_2$ is bounded by (21), it cannot measure the robust tracking performance of platoon directly, which is also affected by $\boldsymbol{X}$. The following theorem gives out an estimation of the upper limit of $\|\boldsymbol{E}(s)\|_2$.

Theorem 1: Under (20), $\|\boldsymbol{E}(s)\|_2$ is bounded by

$$\left\| W_p(s)\,\boldsymbol{E}(s) \right\|_2 \le \frac{\alpha\beta\sqrt{N}}{1-\gamma\beta} \|a_0(s)\|_2 \tag{22}$$

Proof: Substituting (17) and III-C to (19) yields

$$\begin{aligned} &\boldsymbol{E}(s) = \boldsymbol{X}\boldsymbol{T}(s)\,\boldsymbol{X}^{-1}\left[\boldsymbol{W}(s) - \mathbf{1}_N a_0(s)\right] \\ &\boldsymbol{Z}(s) = \Omega(s)\,P(s)\,K(s)\,\boldsymbol{X}\boldsymbol{\Lambda}\boldsymbol{X}^{-1}\boldsymbol{E}(s),\ \boldsymbol{W}(s) = \boldsymbol{\Delta}\boldsymbol{Z}(s) \end{aligned} \tag{23}$$

where $\boldsymbol{T}(s) = diag\left(\boldsymbol{\tau}_d(\boldsymbol{\Lambda}_1), \cdots, \boldsymbol{\tau}_d(\boldsymbol{\Lambda}_n)\right)$. Considering the definition of $\boldsymbol{D}$, (18) and $\|\mathbf{1}_N a_0(s)\|_2 = \sqrt{N}\|a_0(s)\|_2$, (22) can be derived from (23).

Note that the upper limit of $\|\boldsymbol{E}(s)\|_2$ increases with the platoon size and the distance tracking error of each node cannot be uniformly bounded, which is required for string stability. Nevertheless the average energy of distance tracking errors measured by RMS (Root Mean Square) does not increase with platoon size and is limited by:

$$\begin{aligned} \mathrm{RMS}[\boldsymbol{E}(s)] &= \sqrt{\frac{\sum_{i=1}^{N}\int_0^\infty |e_i(j\omega)|^2 \mathrm{d}\omega}{N}} \\ &\le \frac{\alpha\beta}{1-\gamma\beta}\|a_0(s)\|_2 \end{aligned} \tag{24}$$

D. String Stability of Heterogeneous Platoon

Note that the commonly used homogeneous string stability is not applicable to heterogeneous platoons [10]. Here considering the influence of the leader on distance tracking error, the heterogeneous string stability of vehicular platoon is defined as follows

Definition 1 [10]: When the initial state is zero, the platoon (9) is heterogeneous string stable, if for any platoon length N, there exists a constant $\varepsilon > 0$, independent of N, such that

$$\|e_i(s)\|_2 \le \varepsilon\,\|a_0(s)\|_2, \forall i \in \{1, \cdots, N\}. \tag{25}$$

Definition 1 means that if a heterogeneous vehicular platoon is string stable, the propagating errors arising from the leader stay uniformly bounded for all platoon sizes. **Theorem 2** gives a sufficient condition of heterogeneous string stability.

Theorem 2: If $\boldsymbol{P}=\boldsymbol{I}$ and $K(s) = K_s + K_v s + K_a s^2$ satisfies the following condition (26), then the platoon (9) is heterogeneous string stable.

$$\|\Omega(s)\,P(s)\,K(s)\,\boldsymbol{\tau}_d(1)\|_\infty \le \gamma < 1. \tag{26}$$

Proof: Since $\boldsymbol{P}=\boldsymbol{I}$, from **Lemma 3** it is known that its eigenvalues and eigenvectors satisfy (12) and (13). The closed-loop system is derived from (9). Substituting (12) and (13) to it yields

$$\begin{cases} \boldsymbol{T}_{11}(s)\,\bar{X}_1\boldsymbol{E}(s) + \boldsymbol{T}_{12}(s)\,\bar{X}_2\boldsymbol{E}(s) = -\bar{X}_1\mathbf{1}_N a_0(s) \\ \boldsymbol{T}_{21}(s)\,\bar{X}_1\boldsymbol{E}(s) + \boldsymbol{T}_{22}(s)\,\bar{X}_2\boldsymbol{E}(s) = \boldsymbol{O} \end{cases} \tag{27}$$

where $\boldsymbol{T}_{11}(s) = s^2\boldsymbol{I} - P(s)\,K(s)\left[\boldsymbol{I} + \Omega(s)\,\bar{X}_1\boldsymbol{\Delta}\boldsymbol{X}_1\right]$

$$\begin{aligned} \boldsymbol{T}_{21}(s) &= -\Omega(s)\,P(s)\,K(s)\,\bar{X}_2\boldsymbol{\Delta}\boldsymbol{X}_1 \\ \boldsymbol{T}_{12}(s) &= -\Omega(s)\,P(s)\,K(s)\,\bar{X}_1\boldsymbol{\Delta}\boldsymbol{X}_2 \\ \boldsymbol{T}_{22}(s) &= s^2\boldsymbol{I} - P(s)\,K(s)\left[\boldsymbol{I} + \Omega(s)\,\bar{X}_2\boldsymbol{\Delta}\boldsymbol{X}_2\bar{\Lambda}\right]. \end{aligned}$$

The second equation of (27) holds for all possible $\boldsymbol{E}(s)$, if and only if

$$\boldsymbol{T}_{21}(s)\,\bar{X}_1 + \boldsymbol{T}_{22}(s)\,\bar{X}_2 = \boldsymbol{O}. \tag{28}$$

Then the following equation is obtained from (28):

$$\boldsymbol{T}_{21}(s)\,\bar{X}_2 = -\boldsymbol{T}_{12}(s)\,\boldsymbol{T}_{22}^{-1}(s)\,\boldsymbol{T}_{21}(s)\,\bar{X}_1 \tag{29}$$

Multiplying (29) with $\boldsymbol{X}_2\bar{X}_2\boldsymbol{E}(s)$ on both sides and considering the conclusion (c) of **Lemma 3**, we have

$$\begin{aligned} &\boldsymbol{T}_{12}(s)\,\bar{X}_2\boldsymbol{E}(s) \\ &\quad = -\boldsymbol{T}_{12}(s)\,\boldsymbol{T}_{22}^{-1}(s)\,\boldsymbol{T}_{21}(s)\,\bar{X}_1\boldsymbol{X}_2\bar{X}_2\boldsymbol{E}(s) = \boldsymbol{O}. \end{aligned} \tag{30}$$

Multiplying (30) with $\left(\boldsymbol{I} - \boldsymbol{X}_2\bar{X}_2\right)^{-1}\boldsymbol{X}_1$ on both sides and considering $\left(\boldsymbol{I} - \boldsymbol{X}_2\bar{X}_2\right)^{-1}\boldsymbol{X}_1\bar{X}_1 = \boldsymbol{I}$ and $\bar{X}_1\boldsymbol{X}_1 = \boldsymbol{I}$ derived from (13), the following equation is obtained from (30):

$$\boldsymbol{E}(s) = -\boldsymbol{X}_1\boldsymbol{T}_{11}^{-1}(s)\,\bar{X}_1\mathbf{1}_N a_0(s) \tag{31}$$

Since $\bar{X}_1\boldsymbol{X}_1 = \boldsymbol{I}$, $\boldsymbol{T}_{11}(s)$ defined after (27) is rewritten as

$$\boldsymbol{T}_{11}(s) = \bar{X}_1\left[s^2\boldsymbol{I} - P(s)\,K(s)\,\boldsymbol{I} - \Omega(s)\,P(s)\,K(s)\,\boldsymbol{\Delta}\right]\boldsymbol{X}_1. \tag{32}$$

Then we have

$$\boldsymbol{T}_{11}^{-1}(s)\,\bar{X}_1\left[s^2\boldsymbol{I} - P(s)\,K(s)\,\boldsymbol{I} - \Omega(s)\,P(s)\,K(s)\,\boldsymbol{\Delta}\right]\boldsymbol{X}_1 = \boldsymbol{I}. \tag{33}$$

Multiplying (33) with $\boldsymbol{X}_1$ and $\bar{X}_1\left(\boldsymbol{I} - \boldsymbol{X}_2\bar{X}_2\right)^{-1}$ on both sides and substituting $\boldsymbol{X}_1\bar{X}_1\left(\boldsymbol{I} - \boldsymbol{X}_2\bar{X}_2\right)^{-1} = \boldsymbol{I}$ to it yields

$$\boldsymbol{X}_1\boldsymbol{T}_{11}^{-1}(s)\,\bar{X}_1 = \left[s^2\boldsymbol{I} - P(s)\,K(s)\,\boldsymbol{I} - \Omega(s)\,P(s)\,K(s)\,\boldsymbol{\Delta}\right]^{-1}. \tag{34}$$

Substituting (34) to (31), we have

$$\boldsymbol{E}(s) = \left[s^2\boldsymbol{I} - P(s)\,K(s)\,\boldsymbol{I} - \Omega(s)\,P(s)\,K(s)\,\boldsymbol{\Delta}\right]^{-1}\mathbf{1}_N a_0(s). \tag{35}$$

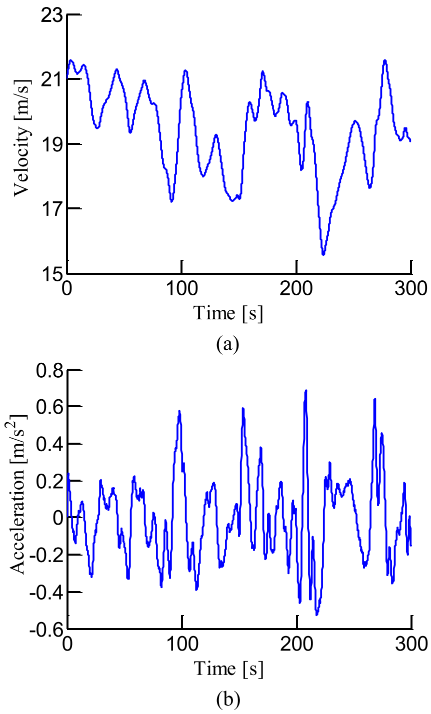

Fig. 4. Profiles of the lead vehicle. (a) Velocity. (b) Acceleration.

From (35), the dynamics coupling among followers arising from communications can be decoupled as

$$e_i(s) = \frac{a_0(s)}{s^2 - P(s)K(s)[1+\Omega(s)\Delta_i]}, i \in \{1, \cdots, N\}. \tag{36}$$

Since $K(s)$ satisfies (26), $K(s)$ can stabilize the uncertain plant $a_i(s) = P(s)[1+\Omega(s)\Delta_i]u_i(s)$. And so the $\boldsymbol{H}_\infty$ norm of the closed-loop system exists and is defined as [28]:

$$\left\| \frac{1}{s^2 - P(s)K(s)[1+\Omega(s)\Delta_i]} \right\|_\infty = \varepsilon \tag{37}$$

Eq. (25) can be obtained from (36) and IV. From **Definition 1**, the platoon is heterogeneous string stable. ■

From the previous studies, it is already know that the leader's information is a necessary condition for string stability when using an identical state-feedback controller [8], [10]. Based on the presented decoupling method, it is further found from **Theorem 2** that if $\boldsymbol{P}$=$\boldsymbol{I}$, the platoon system can be decoupled to N sub-systems essentially, which are independent of the eigen-decomposition of $\boldsymbol{L}$+$\boldsymbol{P}$.

IV. Simulation and Analysis

To validate the effectiveness of the proposed method, a series of simulations were conducted on nonlinear vehicle longitudinal models:

$$\begin{aligned}
\dot{p}_i(t) &= v_i(t), \dot{v}_i(t) = a_i(t) \\
a_i(t) &= \frac{T_i(t)\eta_i}{M_i r_i} - g[f_i\cos(\theta) + \sin(\theta)] \\
&\quad - \frac{0.5C_{d,i}A_i\mathrm{sgn}(v_i(t)+v_w)[v_i(t)+v_w]^2}{M_i} \\
\dot{T}_i(t) &= \left[T_{d,i}(t) - T_i(t)\right]/\tau_i, i = 0, \cdots, N,
\end{aligned} \tag{38}$$

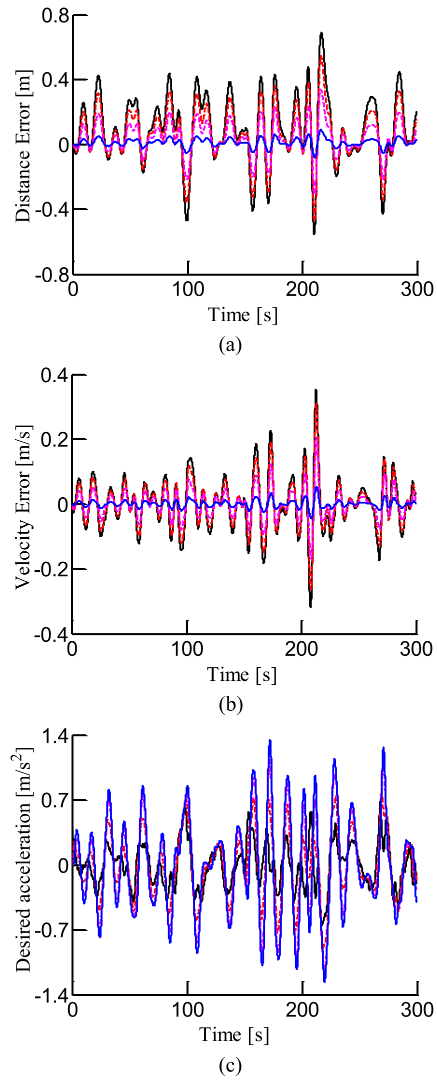

Fig. 5. Simulation results of non-robust controller. (a) Distance tracking error. (b) Velocity tracking error. (c) Desired acceleration.

where $T_i(t)$ and $T_{d,i}(t)$ are the actual and desired driving/brake torques on the wheel, η_i is the lumed mechanical efficiency of driveline, $C_{d,i}$ is the drag coefficient, A_i is the frontal area, r_i is the tire radius, M_i is the vehicle mass, g is the acceleration due to gravity, f_i is the coefficient of rolling resistance, v_w is the wind speed, θ is the road slope, τ_i is the time constant of drivetrain dynamics and sgn (**.**) is the signum function. The nonlinearities of (38) are handled by

$$T_{d,i}(t) = \left[M_{0,i}u_i(t) + C_{d,i}A_i v_i^2(t) + M_{0,i}gf_{0,i}\right] r_i/\eta_{0,i} \tag{39}$$

where $M_{0,i}$, $f_{0,i}$ and $\eta_{0,i}$ are the nominal values of M_i, f_i and η_i, respectively. The simulated platoon includes 1 leader and 10 followers, whose parameters are selected from their varying range. The leader runs according to a naturalistic acceleration/velocity profile, which is from driver experiment data and shown in Fig. 4. The desired distance between two neighboring vehicles is 10 m.

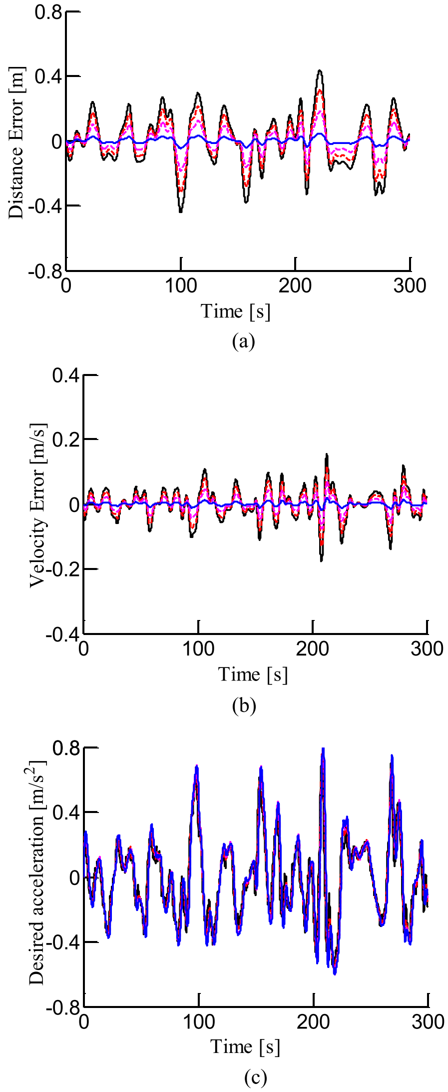

Fig. 6. Simulation results of $\boldsymbol{H}_\infty$ controller. (a) Distance tracking error. (b) Velocity tracking error. (c) Desired acceleration.

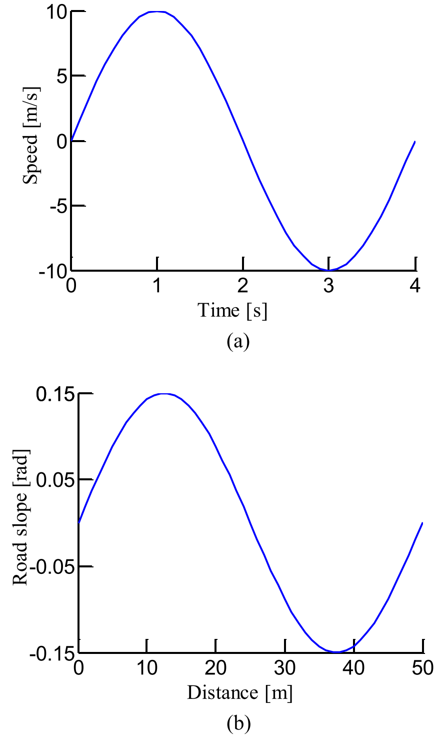

Fig. 7. Profiles of wind speed and road slope. (a) Wind speed. (b) Road slope.

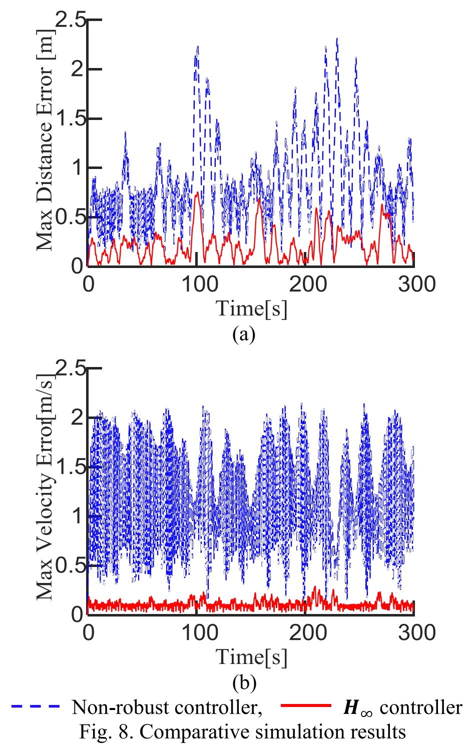

Fig. 8. Comparative simulation results. (a) Distance tracking error. (b) Velocity tracking error.

A. Nominal Condition

A comparative simulation of robust and non-robust controllers has been carried out to show the necessity for considering the robust performance of platoon. The communication topology is BD (Bidirectional) [27]. The non-robust controller is designed to be $K(s) = -8 - 8s - s^2$. The designed $\boldsymbol{H}_\infty$ controller is $K(s) = -10.5 - 38.4s - 12.7s^2$. When all vehicles use their nominal parameters, the simulated results of non-robust and $\boldsymbol{H}_\infty$ controller are shown in Fig. 5 and IV-B respectively. It can be found from Fig. 5 and Fig. 6 that both two controllers can control the vehicular platoon stably with acceptable tracking errors. Comparing with the non-robust controller, the $\boldsymbol{H}_\infty$ controller has a better performance of tracking. The maximum error of distance is reduced from 0.7 m to 0.5 m, and that of velocity is reduced from 0.35 m/s to 0.19 m/s. Comparing 6(c) with the leader's acceleration (Fig. 4(b)), the control input of $\boldsymbol{H}_\infty$ controller is in a reasonable range. Furthermore, a high-gain control law is often needed to compensate for the disturbances in order to achieve smaller control error. By comparing Fig. 5 (c) with Fig. 6(c), it is found that though $\boldsymbol{H}_\infty$ controller has a better disturbance attenuation performance, its control input called desired acceleration is smaller than the non-robust controller. This shows the necessity to optimize the distributed feedback controller by using the $\boldsymbol{H}_\infty$ method.

B. Robust Performance

To further validate the robust performance of designed $\boldsymbol{H}_\infty$ controller, a disturbed condition is used for simulation. The values of vehicle parameters are selected randomly from the possible range. Furthermore, a periodic disturbance arising from both wind and road slope is applied. One cycle of wind speed and road slope is shown in Fig. 7.

Fig. 8 shows the maximum tracking errors under this disturbed condition, from which it is found that the vehicular platoon controlled by the $\boldsymbol{H}_\infty$ controller has a much better performance. Its maximum error of distance and velocity is 0.76 m and 0.3 m/s respectively, while the maximum distance and velocity errors of the non-robust controller are 2.4 m and 2.1 m/s. Comparing with the results in Fig. 6, the maximum distance and velocity errors of $\boldsymbol{H}_\infty$ controller are only increased by 0.26 m and 0.11 m/s respectively, while that of non-robust controller are increased by 1.7 m and 1.75 m/s. This shows that it is necessary to consider the robustness and tracking performance when designing vehicular platoon control system and the proposed distributed $\boldsymbol{H}_\infty$ control strategy successfully balances the performances of robust stability and disturbance attenuation ability. Non-robust controller,

V. Conclusions

This paper presents a decoupled robust control method for a platoon of heterogeneous vehicles with uncertain node dynamics. The following conclusions are obtained: (1) It is necessary to consider the robust stability when synthesizing a vehicular platoon; (2) The proposed decoupled robust synthesis method can ensure the robust stability, tracking performance, and heterogeneous string stability of a platoon; (3) Comparing with the information of followers, the leader information is dominant. If all the followers can receive the leader information, only $\lambda = 1$ needs to be considered when synthesizing the platoon system, which significantly relaxes the requirements of robust stability and tracking performance.

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